NOTES on *Electromagnetic Compatibility C, course run by Sergio Amedeo Pignari*

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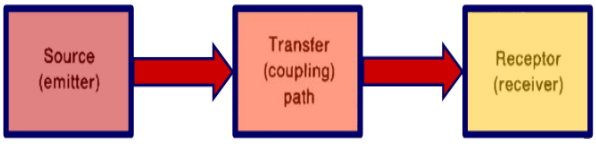
Contents

1 Introduction and Requirements

- **EMI**: Electromagnetic Interference.
- **RFI**: Radio Frequency Interference.
- **EMC** is concerned with undesrtanding the ability of electromagentic emissions to cause interference in electrical and electronic devices; and learing how to design systems for EMC. EMC is defined as the capability of an electrical/electronic system to function compatibly with other electrical/electronic systems and not produce or be susceptible to interference. More precisely it is concerned with the generation, transmission and reception of electromagentic energy. Keep in mind the following blocks scheme:



- **A compatible system**:
  - does not cause interference with other systems
  - it is not susceptible to emission from other systems
  - it does not cause interference with itself
- An important detail: *only undesired behavior of the recepetor constitutes interference*. This means that unintentional transmission or reception of electromagnetic energy is detrimental only if it is of sufficient magnitude and/or spectral content at the receptor. How can we prevent interference?
  - Suppress the emission at its source (firts line of defense)
  - Make the coupling path as inefficient as possible (the efficiency is proportional to the height of the passed signal frequency)
  - Make the receptor less suscepitbe to the emission
- **Susceptibility**: vulnerability of the receptor to electromagentic disturbances.
- Subproblems:
  - Cables: they have the potential for emtting and/or picking up e.m. energy - the longer the cable, the more efficient it is. Moreover, interference signals can be passed directly between the subsystems via diret conduction on cables.
  - Enclosures: we mean metallic enclosures. Currents may be induced on these enclosure by both internal or external signals. These currents can radiate...
  - Radiated Emission (RE) or Radiated Susceptibility (RS): phenomena strictly related to the presence of an accelerated charge.
  - Conducted Emission (CE) and Conducted Susceptibility (CS): as partially suggested by their names, these phenomena occcurs whenever electromagnetic energy is conducted on metallic conductors.
  - CROSSTALK (XTALK): unintended electromagnetic coupling between wires, PCB lands, or IC interconnects that are in close proximity.
  - ESD: electronic discharge. EMP: electromagnetic pulse. Lightninng. EMSEC. Power Quality.

- **Lumped-parameter circuit theory**: circuit elements are treated as ideal, capacitive and/or inductive coupling among different circuit elements/parts are negelected. Propagation effects are not taken into account and circuits are not allowed to radiate or pick up electromagnetic energy.
  Electrical dimensions of the structure are more significant if expressed in wavelength:
  $$d_e = \frac{d_g}{\lambda}$$
  The condition required for lumped-parameter circuit modeling is: $d_e \ll 1$.
  Or:
  $$d_g \ll \lambda \rightarrow \frac{d_g}{\nu} \ll \frac{\lambda}{\nu} \rightarrow T_D \ll T$$

Example 1: we have a lumped-circuit element along its connection lead an we suppose there is a signal which is propagating. In the next tabel it's summarized the analysis of this little simple system.
Note that the critical paremeter is the *electrical length*, that is the ratio between the psysical length and the wavelength of the signal.

Distance:	L
Velocity of signal:	v
Period:	$T_D = \frac{L}{v}$
Propagation constant:	$\beta = \frac{2\pi}{\lambda}$
Fundamental relation:	$\lambda = \frac{v}{f}$
Frequency of signal:	f
Intensity of signal:	$i(z,t) = I \cdot \cos(\omega t - \beta z) = I \cdot \cos\left[\omega \cdot \left(t - \frac{z}{v}\right)\right]$
Phase Shift (z=L):	$\beta L = \frac{2\pi}{\lambda} L = 2\pi \frac{L}{\lambda}$

We can easily understand that if $L \ll \lambda$ the phase shift is negligible!

Example 2: Trasmission line circuit. The electrical lenght determines the voltages (and currents) at the section BB'. Obviously the signal is coming from AA':

$$V_{AA'} = V_0 \cos(\omega t) \quad \longrightarrow \quad V_{BB'} = V_{AA'}\left(t - \frac{L}{c}\right) = V_0 \cos\left[\omega \cdot \left(t - \frac{L}{c}\right)\right]$$

Now we can evaluate how much the length of the transmission line would affect the voltage signal, for istance at $t = 0s$. Another time, the critical parameter is $\omega \frac{L}{c}$, that can be expressed as $2\pi \frac{L}{\lambda}$.

Conclusion: for our purpose we say that a system is *electrically small* when the largest dimension is smaller than one tenth of the smallest wavelength (that is, at maximum frequency). It's known as *rule of thumb*: $L < \lambda_{min}/10$.

- With media that are different from the void, the velocity is not c but "v".
  In the following are shown all the relation to treat with this possibility:

Electricanl Permittivity:	$\epsilon = \epsilon_0 \cdot \epsilon_r$
Magnetic Permeability:	$\mu = \mu_0 \cdot \mu_r$
Constants:	$\epsilon_0 \approx \frac{1}{36\pi} 10^{-9}\ {}^{F}/_{m}$ and $\mu_0 = 4\pi \cdot 10^{-7}\ {}^{H}/_{m}$

- **Requirements**. Two types: mandated by governmental agencies or imposed by product manufacturer.

  The firts ones are called legal requirements and cannot be waived. They are imposed to minimize the elctromagnetic pollution, aka the interference generated by the product. However, compliance with these requirements does not guarantee the absence of interference. Moreover, they must be respected in order to have the product marked.

  The manufacturer requirements, on the other hands, are voluntarily imposed and are intended to result in constumer satisfaction.

  In the US there is the FCC (Federal Communications Commision): it's charged with the regulation of radio e wire communicaation. The range of frquencies the FCC has determined to be "radio frequencies" extends from 9 Khz to 3000 Ghz.
  For what concerns the digital devices: they are defined as unintentional radiators that generate and use timing pulses at a rate in excess of 9000 pulses (cycles) per second...
  The FCC has divided the digital devices into two classes:

  - class A: commercial, industrial or buisness enviroment;
  - class B: residential enviroment

  The frequency range for conducted emissions extends from 150 KHz to 30 MHz.

  The frequency range for radiated emissions extends from 30 MHz to 40 GHz.

2 Practical Lecture: Decibel

The decibel is an important way to express data, because the decibel is a compression of large values or an extension of the small ones. It's defined as a ratio between two quantities, in general:

Voltages:	$20 \log_{10} \left(\frac{V_2}{V_1} \right)$
Currents:	$20 \log_{10} \left(\frac{I_2}{I_1} \right)$
Power:	$10 \log_{10} \left(\frac{P_2}{P_1} \right)$

However, there are problems about the meaning of these values whenever they are meant to be absolutes. To avoid this issue, we will express the previuos physical quantities introducing in the decibel conversion some reference values:

Voltages:	$\text{dBmV} \equiv 20 \log \left(\frac{volts}{1mV} \right)$
Currents:	$\text{dBmA} \equiv 20 \log \left(\frac{amps}{1mA} \right)$
Powers:	$\text{dBmW} \equiv 10 \log \left(\frac{watts}{1mW} \right)$

Moreover, oftenly we need to evaluate the gain of a device. We could be asked to compute the voltage gain or the power gain of an amplifier, for example. Long story short, in decibel the results coincide:

$$G_{p,dB} = 10 \log \left(\frac{P_2}{P_1} \right) = 10 \log \left(\frac{V_2^2/R_2}{V_1^2/R_1} \right) = 20 \log \left(\frac{V_2}{V_1} \right) = G_{v,dB}$$

For what concerns electromagentic compatibility, the most important parameters are fields. Similarly to what seen above:

Electric field:	$dB\mu V/m$
Magnetic field:	$dB\mu A/m$
Impedance:	$dB\Omega$

Moreover we prefer to express voltages and currents in $dB\mu*$.

EMC MEASUREMENT SYSTEMS.
The reference impedance value, at which we want to have matching, is the typical 50Ω.
The signal sources can be modeled by the Thevenin equivalent circuit: V_{OC} and $R_s(\approx 50\Omega)$.
In case of matching:

1. impedance seen at source outlet is constant over frequency and equals to 50Ω
2. no reflections occurs at every matched sections: the received signal is only attenuated due to cable losses
3. the signal source delivers to the load the maximum power:

$$P_{max} = \frac{V_{OC(rms)^2}}{4R_s}$$

This is actually what we should call, more precisely, the *available* power at the the source output. Next, we can show that the output voltage is strictly related to the open circuit voltage - is the voltage that would be measured across a matched load.

$$V_{out} = \frac{R_L}{R_S + R_L} V_{OC} = \frac{1}{2} V_{OC}$$

Relationship between P_{out} and V_{out}:

$$P_{out} = P_{max} = \frac{V_{OC}^2}{4R_S} = \frac{(V_{OC}/2)^2}{R_S} = \frac{V_{out}^2}{R_S}$$

In decibel:

$$P_{out,dBm} = V_{out,dB\mu V} - 107$$

4. Potential Losses: the amplitude of the physical quantity considered decreases esponentially:

$$A(x) = A_0 e^{-\alpha \cdot x}$$

Where α is a coefficient measured in $^{neper}/_{meter}$. Therefore the signal attenuation is given in neper!

$$A_{NEPER} = \log_{10} \frac{A_0}{A_0 e^{-\alpha \cdot x}} = \alpha \cdot x$$

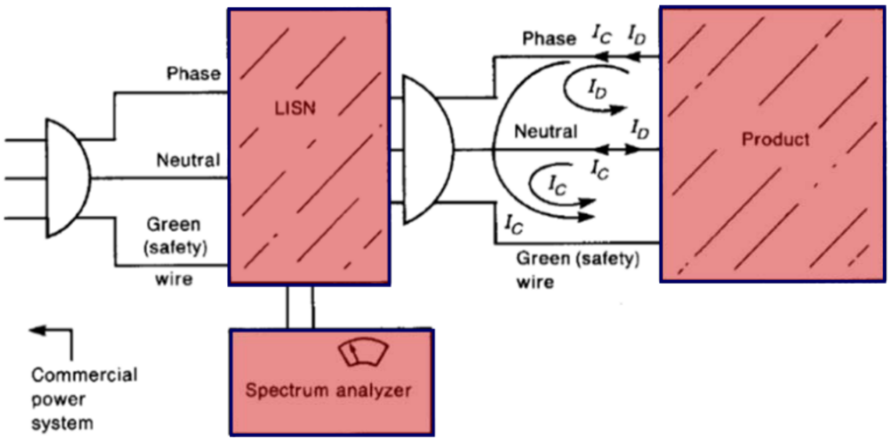
Thus in decibel: $A_{dB} = 8.686 \cdot A_{NEPER}$.

Particular case: Cable losses!
Definition:

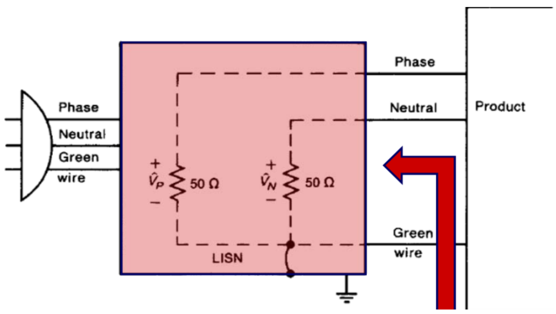
$$\text{Cable Losses} = \frac{P_{in}}{P_{out}} \longrightarrow \text{Cable Losses}_{dBm} = P_{in,dBm} - P_{out,dBm}$$

3 Conducted Emission

- Conducted emissions are noise currents. They are conducted out of the product along the ac power cord. The limits of its range is given in volts: we verify the compliance by measn of LISNs (line impedance stabilization netwrok), placed into the units power cords. The CE are measured via a spectrum analyzer.
- Emission are possibly ascribed to:
  - Switching devices and power electronics components: converters, inverters, variable speed motor drives, etc.
  - Undesired crosstalk (near-field) or radiated emission (far-field)
- **Conducted susceptibility threat**: the CEs exciting a product propagate along its powerline, reaching the power grid and thus other electrical/electronic devices.
- **Radiated susceptibility threat**: the power grid is electrically-large in the CE frequency band, it works like an efficient antenna and giving rise to possible problems in nearby electronic devices.
- **Tyical Test Configuration**

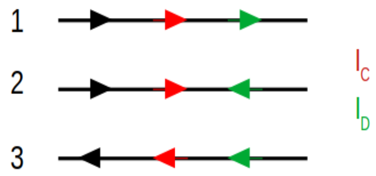


Now, as mentioned before, the LISN is required to control the conducted emissions.
In practice, it's applied to the power cord outlet of the product and its goal is to present a constant impedance over the frequency range of the coduceted emision test: 50Ω between phase conductor and green wire; and 50Ω between neutral conductor and green wires.
The ideal LISN is the following circuit:



- **Modal Decomposition**

  Applied to three-conductors systems.
  Noise currents are separated and related to two configurations: the differential-mode (DM) and the common-mode (CM).
  A possible decomposition is:



$$\begin{cases} \hat{I}_1 = \hat{I}_C + \hat{I}_D \\ \hat{I}_2 = \hat{I}_C - \hat{I}_D \\ \hat{I}_3 = \hat{I}_1 + \hat{I}_2 = 2\hat{I}_C \end{cases}$$

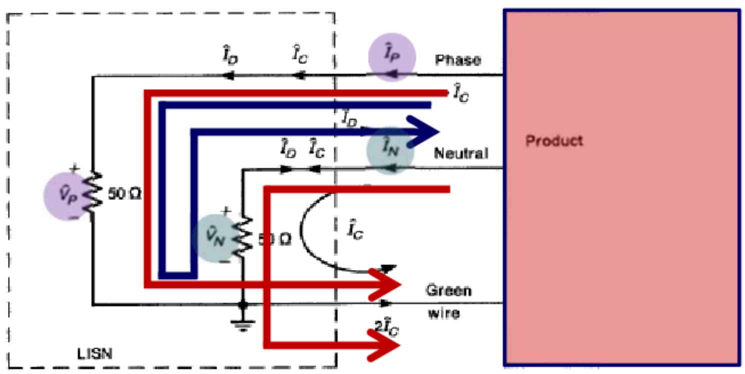
  In the third conductor $\hat{I}_D = 0$. Furthermore we call the third wire the *return conductor*, and:

$\hat{I}_1, \hat{I}_2, \hat{I}_3$	total currents
$\hat{I}_D$ and $\hat{I}_C$	modal components

  Relationnship between modal and common components:

$$\begin{cases} \hat{I}_1 = \hat{I}_C + \hat{I}_D \\ \hat{I}_2 = \hat{I}_C - \hat{I}_D \end{cases} \Rightarrow \begin{cases} \hat{I}_D = \dfrac{\hat{I}_1 - \hat{I}_2}{2} \\ \hat{I}_C = \dfrac{\hat{I}_1 + \hat{I}_2}{2} \end{cases}$$

  Next, relating this theory to the ideal circuit shown above:



$$\begin{cases} \hat{I}_{phase} \leftrightarrow \hat{I}_1 \quad \wedge \quad \hat{I}_{neutral} \leftrightarrow \hat{I}_2 \\ \hat{V}_P = 50 \times (\hat{I}_C + \hat{I}_D) \\ \hat{V}_N = 50 \times (\hat{I}_C - \hat{I}_D) \end{cases}$$

- **Dominant Effect**: to reduce the conducted emissions at a particular frquency we must evaluate and reduce the dominant component. In general we define

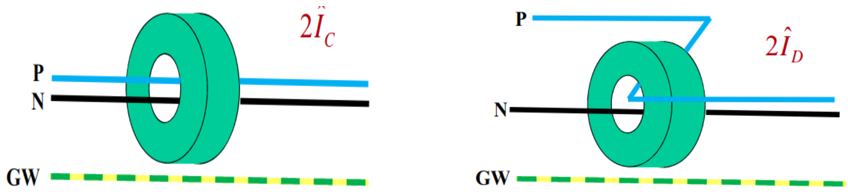
$$I_{total} = I_C \pm I_D$$

  and it's a fucntion of the frequency.

- **Separation of CM and DM**
  Why? Because we must find the causes of emission within the EUT (equipment under test) in order to optimize the power line filter design.
  The diagnostic tool used to achieve this separation is marketed under the label of LISN-UP, invented by C. Paul.
  What we obtain with it?

$$\begin{cases} V_P + V_N = 2 \times 50 I_C = 2V_C \\ V_P - V_N = 2 \times 50 I_D = 2V_D \end{cases} \leftrightarrow \begin{cases} V_P = V_C + V_D \\ V_N = V_C - V_D \end{cases}$$

Just for curiosity, the following pictures show how to configure the wires and probes to do the measurements of the common and differential modes current:



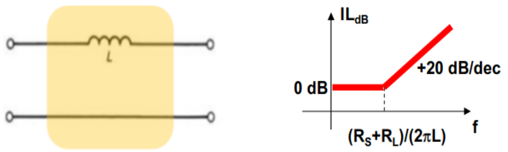
4 Power Supply Filters

Almost all products contain a power supply filter, because usually this is the last circuit through which noise currents pass before going out.
Their task is to reduce CM or DM currents, or both.
Let's begin with the defintion of the insertion loss of the filter:

$$IL_{dB} = 10 \log \left(\frac{P_{L,\text{without filter}}}{P_{L,\text{with filter}}} \right) = 10 \log \left(\frac{V_{L,wo}^2 / R_L}{V_{L,w}^2 / R_L} \right) = 10 \log \left(\frac{V_{L,wo}^2}{V_{L,w}^2} \right)$$

Note that these parameters refer to the magnitudes, they are not complex values.

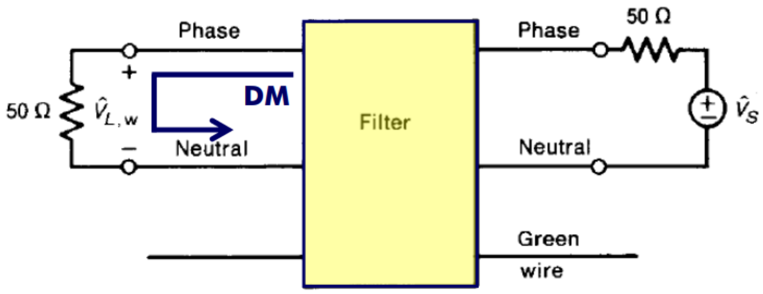
For istancee: Low-pass filter.



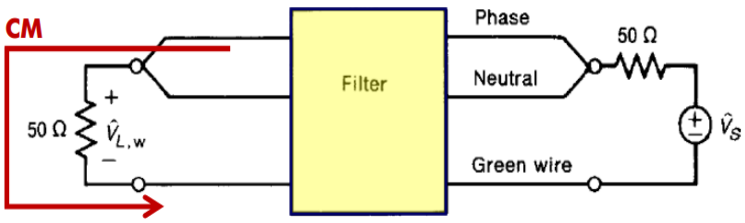
$$IL_{dB} = 10 \log(\omega\tau) \quad \leftrightarrow \quad \tau = \frac{L}{R_S + R_L} \gg 1$$

- **Test set-up**

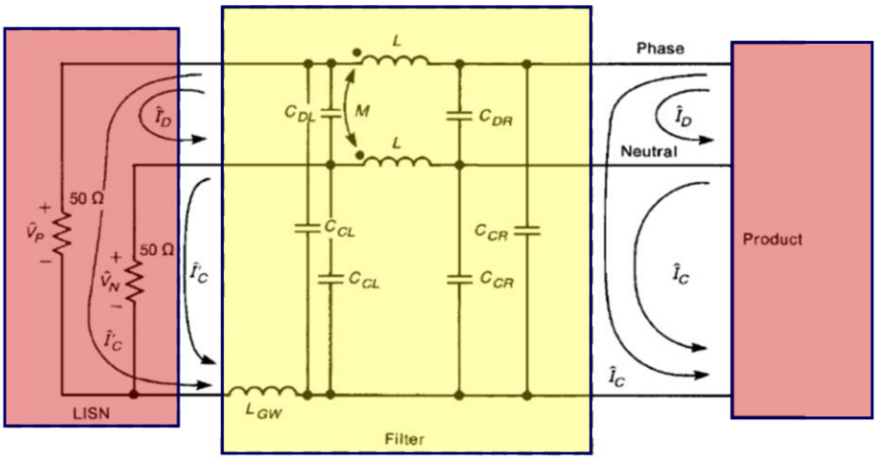
1. DM Insertion Loss measurement: the green wire is left unconnected, while the phase and neutral wires form the circuit to be tested. Sometimes is called also "symmetric mode".



2. CM Insertion Loss measurement: the phase and neutral wires are tied toghether, and the circuit is closed with the green wire. Its alternative name is "asymmetric mode".

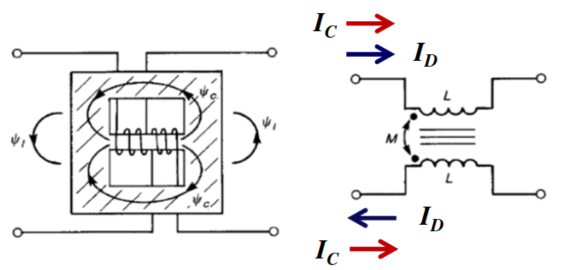


- **The Power Supply Filter topology**



C_{DL}, C_{DR}:	Line-to-line capacitors	to divert DM currents
L, L, M:	Commond-mode Choke	to block CM currents
C_{CL}, C_{CR}:	Line-to-ground capacitors (Y-caps)	to divert CM currents
L_{GW}:	Green-wire inductor	?

- **The Common-Mode Choke**



Physical construction   Equivalent circuit

The windings are indentical, wounded on the same core:

$$L_1 \cong L_2 = L \cong M \rightarrow k = \frac{M}{\sqrt{L_1 \cdot L_2}} \cong \frac{M}{L} \simeq 1$$

We want to compute the voltage-drop across one side of the choke.

What happens with DM current?

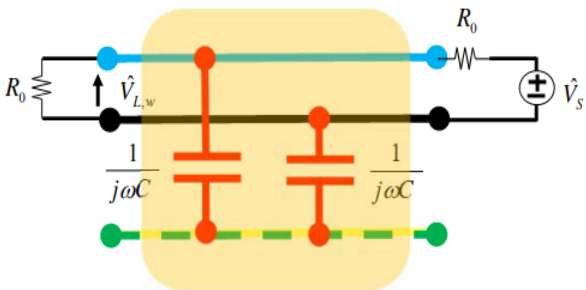
$$\hat{V} = \jmath\omega L\hat{I}_D + \jmath\omega M(-\hat{I}_D) = \jmath\omega(L-M)\hat{I}_D = \jmath\omega L_{leakage}\hat{I}_D \qquad L_{leakage} = 0 \;\; \text{(ideally)}$$

What happens with CM current?

$$\hat{V} = \jmath\omega L\hat{I}_C + \jmath\omega M\hat{I}_C = \jmath\omega(L+M)\hat{I}_C = \jmath\omega L_{leakage}\hat{I}_C$$

In other words we could say that L and M are parameters that gauges the rejection of the choke to current flow, in fact they are add up when we consider CM currents and are substracted with DM currents.
More precisely: in differential mode, the voltage drop across one side of the choke is ideally 0; thus L could be replaced with a short circuit. On the other hand, when the voltage is not 0 and for a current signal that is not constant, the magnetic flux inside the inductances generate an induced current that flows in the opposite direction (law of Faraday-Lenz). Moreover, in the frequency domain we can design the inductor to actually work alike a open circuit in CM mode.

- **Line-to-ground Capacitor**
  Possible shock hazards if one of them shorts out.
  Differential mode IL.

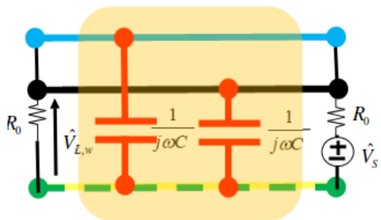


Ideal voltage: $\hat{V}_{L,wo} = \frac{V_s}{2}$

With filtering: $\hat{V}_{L,w} = \hat{V}_s \frac{2R_0}{4R_0 + \jmath\omega C R_0^2}$

Insertion Loss: $IL_{DM}^{dB} = 10\log\left[1 + \left(\frac{\omega R_0 C}{4}\right)^2\right]$

Common mode IL.



After some algebra:

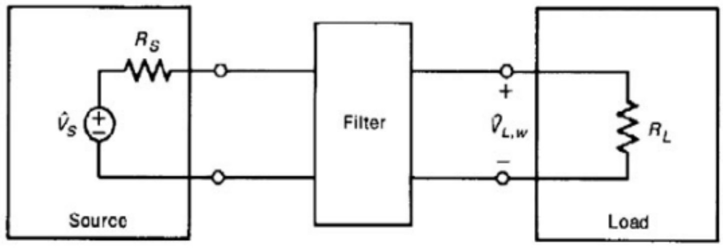
$$IL_{CM}^{dB} = 10\log\left[1 + \omega^2 R_0^2 C^2\right]$$

- **Location of components**: in order to avoid the coupling of power supply/clock harmonics with the wires, the filter should be place as close as possible to the power cord exit and to the power supply.

- **Generic warning**: the inductors are usually connected in series with the wires, and they are specifically used to block currents. They are efficient if their impedance is way higher than load impedance, i.e. they are used in *low impedance circuits*. On the other hand, capacitors are placed in parallel and are used to divert currents. They are employed when the load impedance is higher.

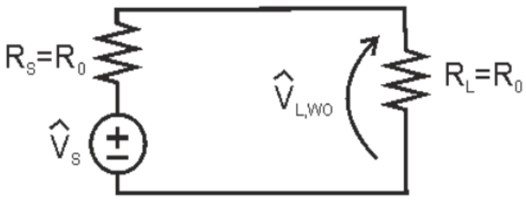
5 Practical Lecture: Insertion Loss Measurement

First part: the general theory



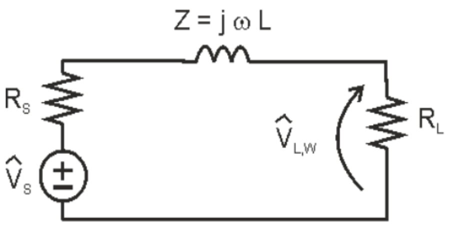
NOISE SOURCE!

- Without filter:



$$\hat{V}_{L,WO} = \frac{\hat{V}_S}{2}$$

- With an inductive filter: used in low-impedance circuits



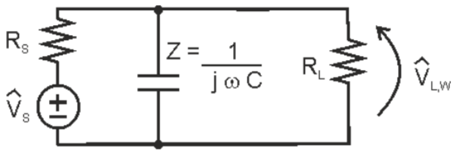
Voltage phasor: $\hat{V}_{L,w} = \frac{R_0}{2R_0 + \jmath\omega L}\hat{V}_s$

Filter IL: $IL_{dB} = 10\log(1+\omega^2\tau^2)$

Time constant: $\tau = \frac{L}{2R_0}$

Cut off frequency: $f_{cutoff} = \frac{1}{2\pi\tau}$

- With a capacitative filter: used in high-impedance circuit



Voltage phasor: $\hat{V} =_{L,w}= \frac{Z_{para}}{Z_{para}+R_0}$

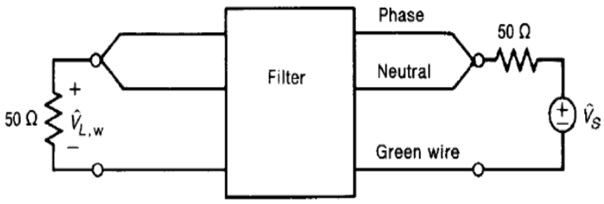
Filter IL: $IL_{dB} = 10\log(1+\omega^2\tau^2)$

Time constant: $\tau = \frac{CR_0}{2}$

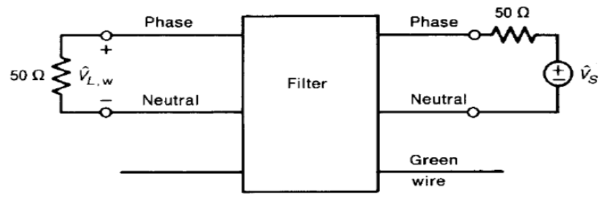
Cut off frequency: $\frac{1}{2\pi\tau}$

Second part: three-paths line of transmissions - modal evaluation

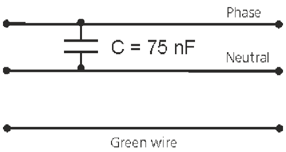
Common Mode



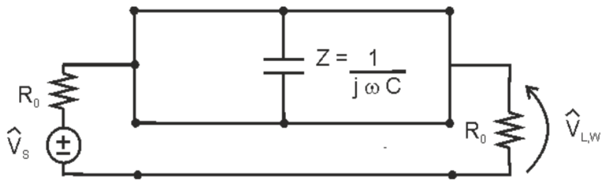
Differential Mode



- Example 1 - a simple powerline (in the place of the filter)



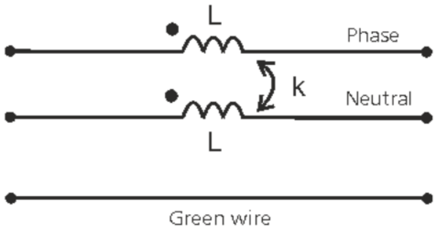
The DM is analyzed with the same reasoning followed for a capacitive filter. On the other hand, in CM, since the circuit topology changes, also the result changes:

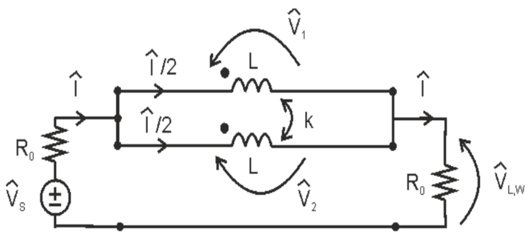


We could easily see that in the end:

$$\hat{V}_{L,w} = \hat{V}_{L,wo} \Rightarrow IL_{dB} = 0$$

- Example 2 - a common-mode choke

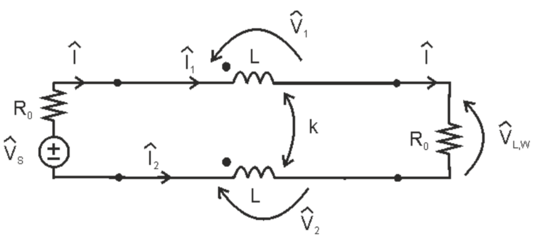




Common Mode:

$$KVL: \quad \hat{V}_s - 2R_0\hat{I} - \jmath\omega(L-M)\frac{\hat{I}}{2} = 0$$

$$\Rightarrow \hat{V}_{L,w} = \frac{2R_0\hat{V}_s}{4R_0 + \jmath\omega(L+M)}$$



Differential Mode:

$$KVL: \quad \hat{V}_s - 2R_0\hat{I} - \underbrace{\jmath\omega(L\hat{I}_1 + M\hat{I}_2)}_{\hat{V}_1} + \underbrace{\jmath\omega(L\hat{I}_2 + M\hat{I}_1)}_{\hat{V}_2} = 0$$

$$\Rightarrow \hat{V}_{L,w} = \frac{R_0\hat{V}_s/2}{R_0 + \jmath\omega(L-M)}$$

Now, we can evaluate the Insertion Losses and the cut-off frequenncies:

Common Mode	$IL_{dB}^{CM} = 10\log_{10}(1+\omega^2\tau^2), \quad \tau = \frac{L+M}{4R_0}, \quad f_{cutoff} = \frac{1}{2\pi\tau} = \frac{2R_0}{\pi(L+M)}$
Differential Mode:	$IL_{dB}^{CM} = 10\log_{10}(1+\omega^2\tau^2), \quad \tau = \frac{L-M}{R_0} = \frac{L_{leak}}{R_0}, \quad f_{cutoff} = \frac{2R_0}{2\pi L_{leak}}$

Note that, because of the leakage inductance in differential mode, the common mode choke would start reduce also DM currents once the frequency increases over its cut-off frequency.

6 Non ideal behaviour

- **Modelling component leads**:
  1. Inductance of attachment leads:

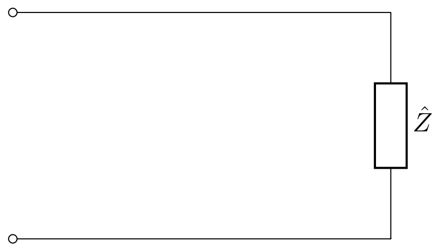
Separation:	s
Wires Radius:	rw
Wires Length:	$\mathcal{L}$
Inductance:	$L_{lead} = \frac{\mu_0}{4} \ln\left(\frac{s}{rw}\right) \cdot \mathcal{L}$

  2. Capacitance of attachment leads:

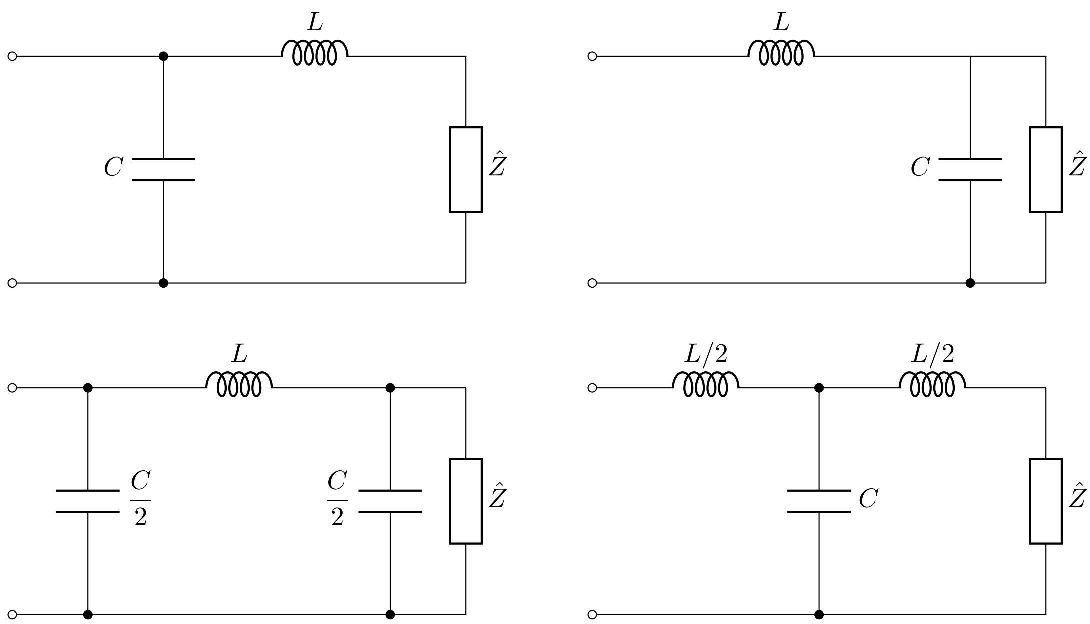
Separation:	s
Wires Radius:	rw
Wires Length:	$\mathcal{L}$
Capacitance:	$C_{lead} = \frac{\pi \epsilon_0}{\ln(s/rw)} \cdot \mathcal{L}$

- How we can take into account both the effects? Well, there is not a unique answer. We usually rely on the lumped components equivalence theory, but there is not a single model: it's a distributed parameter phenomenon!

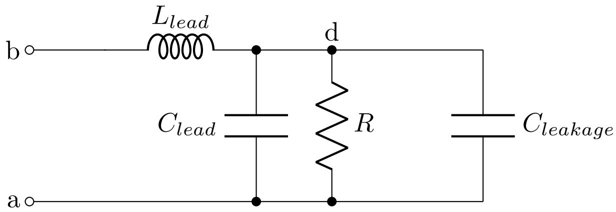
Reference circuit:



Possible solutions:

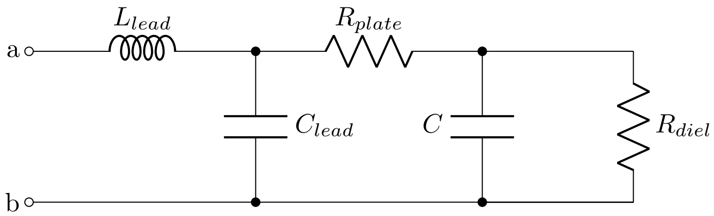


- **Resistors**: the behaviour depends on the construction technique. Wire wound resistor behaves more like inductors, while wire wound and carbon composition resistor behave like capacitors. The resistor ideal impedance is real: $Z_{res} = R + j0$. The bode plots show that both amplitude and phase are constant. For istance, neglecting the parasitic inductance of a non-ideal resistor, the previous situation would look like this:



$$C_{paras} = C_{lead} + C_{leakage}$$

- **Capacitors**: to suppress EMC we can use tantalum electrolytic capacitor for CE a or ceramic capacitors for radiated emissions. The ideal impedance of a capcitor is pure imaginary: $Z = 0 + (1/\jmath\omega C)$. In frequency domain, the capacitors effects are an amplitude decrease by 20dB/dec and a phase change of -90°.
The circuit with a capacitor:



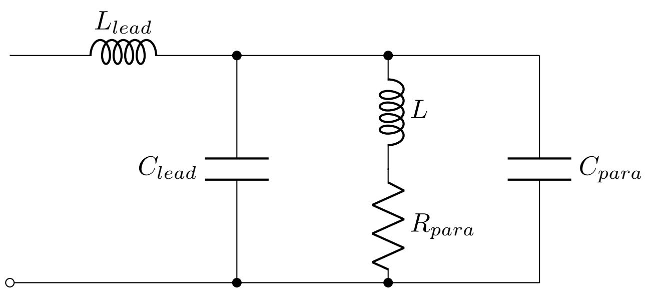
Usually R_{diel} and C_{lead} can be neglected. The total non ideal equivalent impedance is determined by the series of L_{lead}, R_{plate} and C. Besides, R_{plate} is known also as Equivalent Series Resistance (ESR $\rightarrow R_s$:

$$Z_{equivalent} = R + j\left(\frac{\omega^2 CL - 1}{\omega C}\right) \quad \Rightarrow \quad |Z_{equivalent}| = \sqrt{R^2 + \left(\frac{\omega^2 CL - 1}{\omega C}\right)^2}$$

It's evident that for $\omega \rightarrow 0 : |Z| \rightarrow +\infty$. In other words, at low frequencies the dominant contribution is provided by the capacitor (the inductor behaves like a short circuit)
On the other hand, for $\omega \rightarrow \infty, |Z| \rightarrow +\infty$: at high frequencies is the inductor that prevails, the roles are switched.
Note that minimum of the |Z| function is also the exact point at which the capacitor and inductor switch their roles. The corresponding frequency is called *self-resonant* frequency of the capacitor:

$$f_0 = \frac{1}{2\pi\sqrt{L_{lead}C}}$$

- **Inductors**: as for the other devices, the contruction technique determine the non-ideality. In general the process of winding turns of wire on a cylindrical form introduces resistance of the wire and capacitance between neighboring turns. The ideal model has an impedance value that is pure imaginary: $Z = \jmath\omega L$. In the Bode plot, likewise we'se demonstrate for capcitors, the inductors affect both amplitude and phase: amplitude increases by 20dB/dec and phase changes by $+90°$.



Neglecting C_{lead} and L_{lead}, the analysis leads to

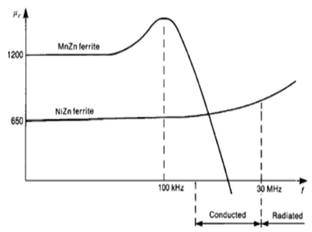
$$f_0 = \frac{1}{2\pi\sqrt{LC_{para}}}$$

that is called *self-resonant* frequency of the inductor.

- **Ferromagnetic Materials**: they are used as core of inductors to increase their inductance. Are widely used in EMC for noise suppression. The important properties are:
  - The ability to concentrate magnetic flux. It stems from the large relative permeability, although this latter parameter depends on magnetic field intensity (H, saturation), as well as on frequency. Remember that the range of interest in EMC is quite large...
  - Saturation: the inductance decreases wth increasing current:

$$\mu = \frac{\Delta B}{\Delta H} \qquad L \propto \mu$$

  - Frequency response:



  As we can see (badly) from the picture, the mainly used materials are ferrites. They are basically non conductive ceramic materials. The most common form is a bead which can be inserted in series with a wire or land, and provide a high frequency impedance in that conductor. In EMC they are used to provide a seletive attentuation of high-frequency noise that we may wish to suppress.

- Ferrite beads.

Inductance:	$L_{bead} = \mu_0 \mu_r K$
Dimensional paramter:	K
Complex permeability:	$\hat{\mu}_r = \hat{\mu}_r{}'(f) - \jmath \hat{\mu}_r{}''(f)$

The real part is related to the stored magentic energy, while the imaginary part is related to the losses. As for the impedance of beads:

$$Z = \jmath\omega L_{bead}(f) = \jmath\omega\mu_0\hat{\mu}_r(f)K = \jmath\omega\mu_0[\hat{\mu}_r{}'(f) - \jmath\hat{\mu}_r{}''(f)]K$$

Hence:

$$Z = \underbrace{\omega\mu_0\hat{\mu}_r{}''(f)K}_{R(f)} + \jmath\omega\underbrace{\mu_0 m\hat{u}_r{}'(f)K}_{L(f)}$$

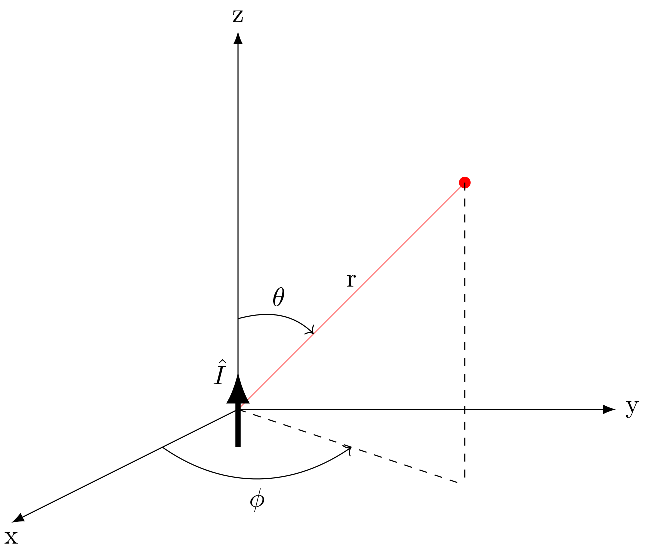
7 Radiated Emissions

- **Radiated emissions**: electromagnetic emissions radiated by the product in the surrunding environment. RE are produced by currents.

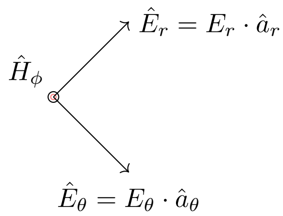
Class A	digital devices used in commercial, industrial or business environment
Class B	digital devices use in residential environment

Keep in mind that for Class B products, RE must be limited to the CISPR22 requirement: from 30MHz up to over 1GHz.
Moreover, the distance at which the measurements must be done is 10m. Indeed, at this value we have the product in the near-field region at low frequencies and in the far-field region for higher intervals.

- **Hertzian Dipole**



The thick arrow placed in the origin represents the hertzian dipole, an *infinitesimal* long antenna in which a current ($\hat{I}$ in phasorial expression) flows in the direction of the arrow. At the red point the vector representation of the field components, on the θ plane, is:



- Now we will observe the expression of the components in near-field and far-field regions. Do recall the intrinsic impedance of free space and the propagation constant:

$$\eta_0 = \sqrt{\mu_0/\epsilon_0} \simeq 377\Omega \qquad \beta_0 = \frac{2\pi}{\lambda_0} \wedge \lambda_0 = \frac{c}{\nu_0}$$

Near field region:

$$\hat{E}_r = 2\frac{\hat{I} \cdot dl}{4\pi}\eta_0\beta_0^2 \cos\theta \left(\frac{1}{\beta_0^2 r^2} - \jmath\frac{1}{\beta_0^3 r^3}\right) e^{-\jmath\beta_0 r}$$

$$\hat{E}_\theta = \frac{\hat{I} \cdot dl}{4\pi}\eta_0\beta_0^2 \sin\theta \left(\jmath\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - \jmath\frac{1}{\beta_0^3 r^3}\right) e^{-\jmath\beta_0 r}$$

$$\hat{E}_\phi = 0$$

$$\hat{H}_r = 0$$

$$\hat{H}_\theta = 0$$

$$\hat{H}_\phi = \frac{\hat{I} \cdot dl}{4\pi}\beta_0^2 \sin\theta \left(\jmath\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2}\right) e^{-\jmath\beta_0 r}$$

In the following are shown the explicit expressions of the functions in the Far field region, reached over the point at which the term $\frac{1}{r}$ becomes dominant. For the Hertzian dipole: $r_B = {}^{\lambda_0}/_{2\pi} \simeq {}^{\lambda_0}/_6$.

$$\begin{cases} \hat{E}_r \to 0 \\ \hat{E}_\theta = \frac{\hat{I} \cdot dl}{4\pi} \eta_0 \beta_0^2 \sin\theta \left(\jmath \frac{1}{\beta_0 r} \right) e^{-\jmath\beta_0 r} \\ \hat{H}_\phi = \frac{\hat{I} \cdot dl}{4\pi} \beta_0^2 \sin\theta \left(\jmath \frac{1}{\beta_0 r} \right) e^{-\jmath\beta_0 r} \end{cases} \Rightarrow \begin{cases} \vec{E}_{far-field} = \jmath\beta_0\eta_0 \frac{\hat{I} \cdot dl}{4\pi} \sin\theta \frac{e^{-\jmath\beta_0 r}}{r} \bar{a}_\theta \\ \vec{H}_{far-field} = \jmath\beta_0 \frac{\hat{I} \cdot dl}{4\pi} \sin\theta \frac{e^{-\jmath\beta_0 r}}{r} \bar{a}_\phi \end{cases}$$

1. the fields are proportional to ${}^1/_r$, I, dl, $\sin\theta$
2. E and H are locally orthogonal
3. $\frac{|\vec{E}|}{|\vec{H}|} = \eta_0$
4. $\vec{E} \times \vec{H} \parallel \bar{a}_r$
5. in the time domain the phase term $exp(-\jmath\beta_0 r)$ traslates to a time delay of $\cos[\omega_0(t - r/\nu_0)]$

- **Inverse Distance Rule**: from the proportionality to ${}^1/_r$ is derived that:

$$|\vec{E}_{D2}| = \frac{D1}{D2} |\vec{E}_{D1}|$$

- **General expression**, valid also for other wire-type antennas

$$\hat{E}_\theta = \hat{M}\hat{I} \frac{e^{-\jmath\beta_0 r}}{r} F(\theta), \qquad \hat{H}_\phi = \frac{\hat{E}_\theta}{\eta_0}$$

F(θ) is referred to as the antenna radiation pattern. For a Hertzian Dipole is equal to $\sin\theta$, in 3D it's a doghnut around the antenna. Moreover:

$$\hat{M} = \jmath \frac{\eta_0 \beta_0 dl}{4\pi} = \jmath 2\pi 10^{-7} f dl$$

- **DM and CM currents** of two parallel conductors:

$$\begin{cases} \hat{I}_1 = \hat{I}_C + \hat{I}_D \\ \hat{I}_2 = \hat{I}_C - \hat{I}_D \end{cases} \Rightarrow \begin{cases} \hat{I}_D = \frac{\hat{I}_1 - \hat{I}_2}{2} \\ \hat{I}_C = \frac{\hat{I}_1 + \hat{I}_2}{2} \end{cases}$$

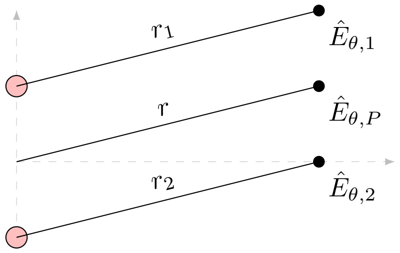
DM currents are predicted by the majority of circuits, they are functional and usually higher than CM currents. Moreover CM currents give rise to larger radiated emissions.
Note that, if we consider only infinitesimally long segments of a trasmission line, we have two approximate hertzian dipoles!
Keeping this in mind, it's evident that the RE of DM currents are 0 at any point equidistant from the radiating antennas: the currents have opposite directions, so the fields emitted are opposite as well. On the other hand, the components of each CM current are directed in the same way: they will add.

- **Total radiated field** in the far-field region for two parallel wire-type antennas, considering components only on the θ plane is:

$$\hat{E}_\theta = \hat{E}_{\theta,1} + \hat{E}_{\theta,2} \qquad \hat{E}_{\theta,i} = \hat{M}\hat{I}_i \frac{e^{-j\beta_0 r_i}}{r_i}$$

It's easier to evaluate the fields with respect to the midpoint between the wires:

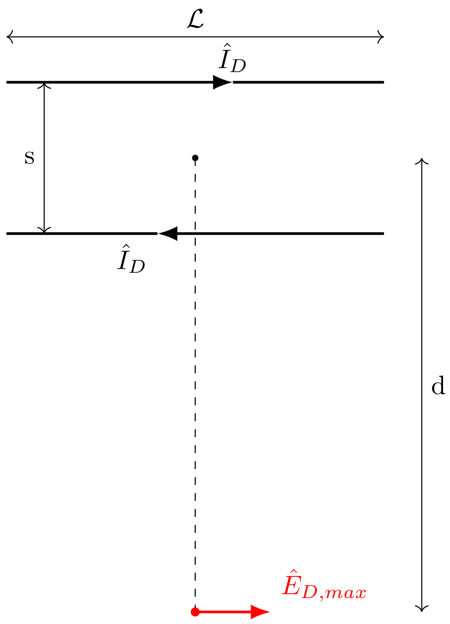
$$r_1 = r - \frac{s}{2}\cos\phi$$
$$r_2 = r + \frac{s}{2}\cos\phi$$



However in the dominator we will trust the following approximation: $r_1 \cong r_2 \cong r$. Hence, we obtain:

$$\hat{E}_\theta = \hat{M}\frac{e^{-j\beta_0 r}}{r}\left(\hat{I}_1 e^{+j\beta_0 \frac{s}{2}\cos\phi} + \hat{I}_2 e^{-j\beta_0 \frac{s}{2}\cos\phi}\right)$$

- **For DM currents**:



$$\hat{I}_1 = \hat{I}_D \quad \wedge \quad \hat{I}_2 = -\hat{I}_D$$
$$r = d \qquad \phi = 0$$

The spacing "s" is electrically small, so $s/\lambda \ll 1$. In this way we can use this approximation:

$$\frac{1}{2}\beta_0 s = \frac{1}{2}\frac{2\pi}{\lambda}s = \pi\frac{s}{\lambda} \ll 1 \Rightarrow \sin\left(\frac{1}{2}\beta_0 s\right) \cong \frac{1}{2}\beta_0 s$$

Hence, applying the theory explained above we obtain a general expression:

$$\hat{E}_{D,max} = j2\pi \times 10^{-7}\frac{f\hat{I}_D\mathcal{L}}{d}e^{-j\beta_0 d}\left\{e^{+j\beta_0 s/2} - e^{-j\beta_0 s/2}\right\}$$

But taking into account the approximation:

$$\hat{E}_{D,max} = -4\pi \times 10^{-7}\frac{f\hat{I}_D\mathcal{L}}{d}e^{-j\beta_0 d}\underbrace{\sin\left(\frac{1}{2}\beta_0 s\right)}_{\to\frac{1}{2}\beta_0 s}$$

Next, the magnitude:

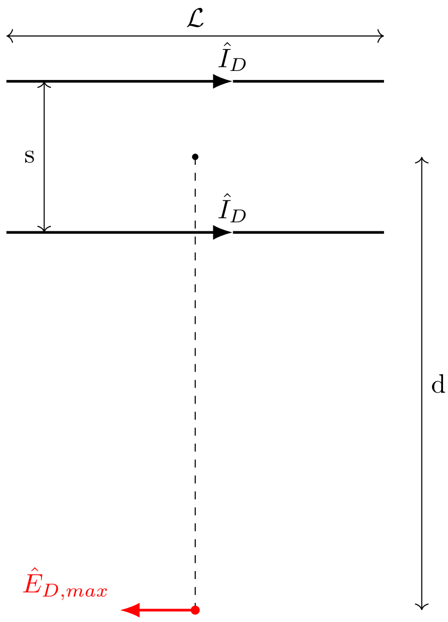
$$\boxed{|\hat{E}_{D,max}| = 1.316 \times 10^{-14}\frac{|\hat{I}_D|f^2\mathcal{L}s}{d}}$$

It's really important to understand that the superposition theorem has been exploited, but the result is not 0 because the two wires are not equidistant from the point at which we evaluated the electric field. However, changing the point also the distances r_1 and r_2 will change.
The total electric field is sensitive to rotation of the cable.

- **Reduction strategies**.
Firstly, we need to mention that significant level of electric field are generally measured at high frequency, typically above 200MHz. So, when we desire to reduce RE at a specific frequency, we could either reduce the current level, either reduce the *loop area* ($A = \mathcal{L} \cdot s$).

The first option is efficiently achieved increasing the rise/fall-times of the current pulse-signal, or decreasing the pulse repetition rate $(1/T)$.
THe second one, on the other hand, should be addressed early in the design.

- **For CM currents**:



$$\hat{I}_1 = \hat{I}_2 = I_D$$

$$r = d \qquad \phi = 0$$

Similarly to what we'done before:

$$\frac{1}{2}\beta_0 s \cong \frac{1}{2} \times \frac{2\pi}{\lambda} s = \pi \frac{s}{\lambda} \ll 1 \quad \Rightarrow \quad \cos\left(\frac{1}{2}\beta_0 s\right) \cong 1$$

And finally we end up with:

$$\hat{E}_{CM,max} = \jmath 4\pi \times 10^{-7} \frac{f \hat{I}_c \mathcal{L}}{d} e^{-\jmath\beta_0 d} \cos\left(\frac{1}{2}\beta_0 s\right)$$

Next we calculate the magnitude:

$$\boxed{|\hat{E}_{CM,max}| = 1.257 \times 10^{-6} \frac{|\hat{I}_c| f \mathcal{L}}{d}}$$

For electical small wire separation the pattern is virtually omnidirectional: the field generated by CM currents is not sensitive to rotation of the cables. In other words we could replace the wires with only one carrying $2\hat{I}_c$.

- **Reduction Strategies**.
  At a specific frequency, we reduce Re reducing the current level or the line length. The first option is achieved by reducing the peak levels, A; or increasing the pulse rise/falltimes (τ_r) or decreasing the pulse repetition rate (1/T).The second strategy should be addressed early in the design, and tends to be more f a problem with wiring hearness.

- **Field Regions**.
  1. *Reactive near-field region.* It's the portion of the near-field immediately surroinding the antenna wherein reactive fields are predominant. In this region, E and H are out of phase by 90 degrees to each other:
  $$\begin{cases} \vec{E} = \hat{E} \cdot \vec{a}_E & \hat{E} = |\hat{E}| e^{\jmath\theta_E}; \\ \vec{H} = \hat{H} \cdot \vec{a}_H & \hat{H} = |\hat{H}| e^{\jmath\theta_H} = |\hat{H}| e^{\jmath(\theta_E + \pi/2)} \end{cases}$$
  Therefore the Poyinting vector is nill in this region.
  2. *Radiating near-field (Fresnel) region.* In this portion of the near-field region the radiation fields start begin to emerge. Differently from the far-field region, here the shape of the radiation pattern may vary appreciably with distance.
  3. *Far-fields (Fraunhofer) region.* In this region the angular field distribution is essentially independent of the distance from the antenna. Obviously, the radiated fields are predominant. Besides, we are far enough to neglect size and shape of antennas. We can also assume that the e.m. waves are purely a radiating spherical wave, locally plane. Lookin at their explicit expression in the case of an ideal Hertzian Dipole, the field components decay as $1/r$.

- **Far-field region**. The boundaries can approximately set at:

$\lambda_0/(2\pi)$	Hertzian Dipole
$3\lambda_0$	"Wire-type" antennas
$2D^2/\lambda_0$	"Surface-type" antennas

Note that in communication systems, the receivers are always in the far-fields of the transmitting antennas. However, the RE that cause interference are due to sources for which the receiver is in the near-field region. Therefore, since the boundaries vary with frequency, keep in mind that changing for istance form 1GHz to 30MHz would probably change the region even if the receiver is still in the same position!

- **Average power density**. The total average power emitted is computed as the integral of the average power density vector over a suitable closed surface surrounding the antenna.
For a Hertzian Dipole:

$$\vec{S}_{ave} = \frac{1}{2}Re\{\vec{E} \times \vec{H}^*\} = \frac{1}{2}Re\{(\hat{E}_\theta \vec{a}_\theta + \hat{E}_r \vec{a}_r) \times \hat{H}_\phi^* \vec{a}_\phi\} = \frac{1}{2}Re\{\hat{E}_\theta \hat{H}_\phi^* \vec{a}_r - \hat{E}_r \hat{H}_\phi^* \vec{a}_\theta\}$$

Without showin every mathematical passage:

$$\hat{E}_r \hat{H}_\phi^* \longrightarrow 0$$

$$\hat{E}_\theta \hat{H}_\phi^* \longrightarrow \frac{1}{(\beta_0 \cdot r)^2}$$

Hence:

$$\vec{S}_{ave} = S(r,\theta) \cdot \vec{a}_r$$

Where:

$$S(r,\theta) = 15\pi \left(\frac{dl}{\lambda_0}\right)^2 |\hat{I}|^2 \frac{\sin^2\theta}{r^2} = S_0 \sin^2\theta$$

And:

$$S_0 = S_{max} = 15\pi \left(\frac{dl}{\lambda_0}\right)^2 |\hat{I}|^2 \frac{1}{r^2}$$

Now, supposing to have a hertzian dipole in the origin of an observating sphere.
The differential power radiated through an elemental area dA is:

$$dP_{rad} = \vec{S}_{ave} \cdot d\vec{A} = \vec{S}_{ave} \cdot \vec{a}_r dA = S_{ave} \cdot dA$$

In the far field region of any antenna, S_{ave} is always in the axil direction.
The spherical coordinate system is recalled:

$$dA = r^2 \sin\theta d\theta d\phi, \qquad d\Omega = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

So we have:

$$dP_{rad} = S_{ave} dA = r^2 S_{ave}(r,\theta,\phi) d\Omega$$

The total radiated power (for a Hertzian dipole) is

$$\boxed{P_{rad} = \int_\Omega dP_{rad} = r^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S_{av}(r,\theta,\phi) \sin\theta d\theta d\phi = \{\cdots\} = 80\pi^2 \left(\frac{dl}{\lambda_0}\right)^2 \frac{|\hat{I}|^2}{2}}$$

- **Radiation resistance**.

$$P_{rad} = \frac{1}{2}|\hat{I}|^2 R_{rad} \quad \Rightarrow \quad R_{rad}$$

It's a fictitious resistance. The larger the radiation resistance, the higher the antenna effectiveness as a radiator.
For the Hertzian Dipole:

$$R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda_0}\right)^2$$

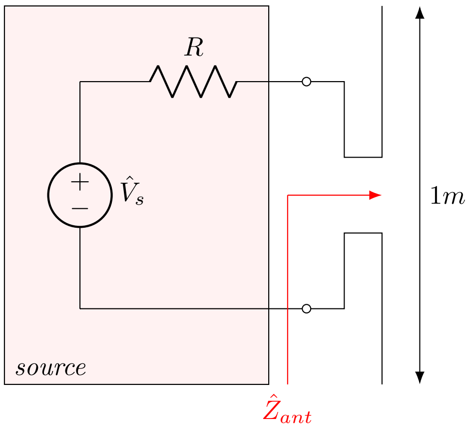
Actually, the Hertzian Dipole is a very inefficient radiator.

- **Input Impedance** is the next step. The total impedance seen at the terminals of the dipole antenna:

$$\hat{Z}_{ant} = R_{in} + \jmath X_{in} = R_{loss} + R_{rad} + \jmath X_{in}$$

For monopoles that are shorter than one-quarter wavelength (or dipoles shorter than one-half wavelength) the reactive part becomes negative, symbolizing a capacitative reactance. For a dipole that is shorter than one-half wavelength is zero.

- **Antenna equivalent circuit**:



Namely:

$$\hat{I}_{ant} = \frac{\hat{V}_s}{R_s + \hat{Z}_{ant}} = \frac{\hat{V}_s}{R_s + R_{loss} + R_{rad} + \jmath X_{in}}$$

$$P_{rad} = \frac{1}{2}|\hat{I}_{ant}|^2 R_{rad}$$

$$P_{loss} = \frac{1}{2}|\hat{I}_{ant}|^2 R_{loss}$$

$$P_{ant} = P_{rad} + P_{loss}$$

- **Antenna Factor**: a parameter defined as the ratio of the electric field at the surfce of the measurement antenna to the received voltage at the antenna terminals:

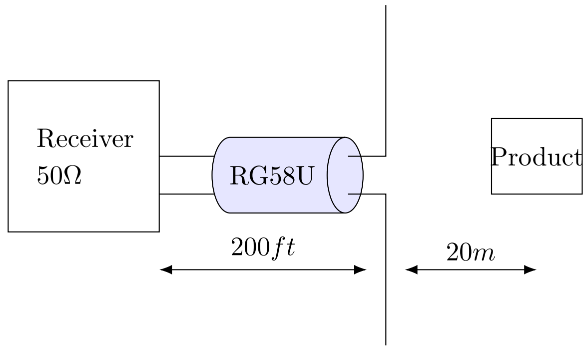
$$AF = \frac{|\hat{E}_{inc}|}{|\hat{V}_{rec}|}$$

In decibel:

$$AF_{dBm^{-1}} = E^{inc}_{dB\mu V/m} - V^{rec}_{dB\mu V}$$

8 Practical Lecture: Radiated Emission Models

- Problem no. 1: *An antenna measures the radiated emissions at 200 MHz from a product as shown in the figure below. If the receiver measures a level of* -93.5 *dBm at 220 MHz, determine the voltage at the base of the antenna in dBμV. The cable loss at 200MHz is 8 dB/100 ft. If the product providing these emissions is located a distance of 20 m and the antenna provides 1.5 V for every V/m of incident field at 220 MHz, determine whether the emissions comply with the CISPR 22 Class B and FCC Class B limits and by how much.*



Solution:

Keep in mind that dBm is a unit of measure for power values, **not** for voltages. Remind that there is a simple relationship to convert the measurement from power to voltage, or viceversa:

$$P_{dBm} = V_{dB\mu V} - 107 \Rightarrow V_{dB\mu V} = -93.5 \;\; dBm + 107 = 13.5 \;\; dB\mu V$$

Voltage at the base of the antenna:

$$V^{ant}_{dB\mu V} = V^{rec}_{dB\mu V} + Cable\ Losses \Rightarrow 13.5 \;\; dB\mu V + \frac{8dB}{100ft} \times 200ft = 29.5 \;\; dB\mu V$$

Now, observe that the expression "*providing 1.5 V for every V/m*" is a clear reference to the antenna factor, which is defined as the ratio between the magnitudes of the voltage detected by the antenna, and the supposed electric field that radiate it.

$$AF = \frac{|\hat{E}_{inc}|}{|\hat{V}_{ant}|} \Rightarrow AF = \frac{1V/m}{1.5V} \rightarrow AF_{dB/m} = -3.52 \;\; dB/m$$

Finally:

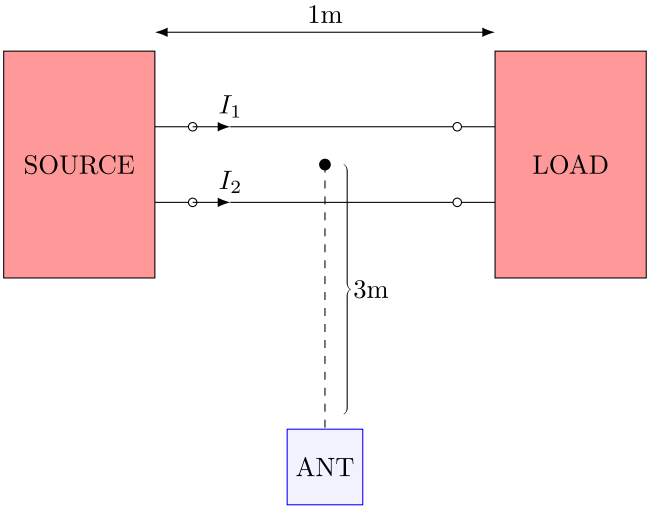
$$E^{inc}_{dB\mu V/m}(20m) = V^{ant}_{dB\mu V} + AF_{dB/m} = 25.98 \;\; dB\mu V/m$$

However we have to evaluate the field at 3m and 10m in order to do the comparison to the limits required:

$$E_{dB\mu V/m}(3m) = E^{inc}_{dB\mu V/m}(20m) + 20 \cdot \log_{10}\left(\tfrac{20}{3}\right) = 42.5 \;\; dB\mu V/m$$ FCC TEST PASSED

$$E_{dB\mu V/m}(10m) = E^{inc}_{dB\mu V/m}(20m) + 20 \cdot \log_{10}\left(\tfrac{10}{3}\right) = 32 \;\; dB\mu V/m$$ CISPR TEST NOT PASSED

- <u>Problem no.2</u>: *The radiated emissions of a cable are being measured as shown at 100 MHz. Determine the magnitudes of the voltages corresponding to the maximum electric field due to the differential-mode and common-mode component measured by the spectrum analyzer if the antenna factor at 100Mhz is 15 dBm*$^{-1}$ *an the antenna is oriented parallel to and in the plane of the wires. The currents values are* $I_1 = 100 \ \ mA$ *and* $I_2 = 10 \ \ mA$*. The spacing is 10 cm.*



<u>Solution</u>:

1. Since the currents in the wires are directed in the same verse the definition of the modal currents are the following:

$$\begin{cases} \hat{I}_D = \dfrac{\hat{I}_1 - \hat{I}_2}{2} \\ \\ \hat{I}_C = \dfrac{\hat{I}_1 + \hat{I}_2}{2} \end{cases} \Rightarrow \begin{cases} \hat{I}_D = 45 \ \ mA \\ \\ \hat{I}_C = 55 \ \ mA \end{cases}$$

2. At 100 MHz we have a wavelength of $c/f = 3m$, so we can assume the spacing to be electrically small. Applying the thoery:

$$|\hat{E}_{DM,max}| = 1.316 \cdot 10^{-14} \cdot \frac{|\hat{I}_D| f^2 \mathcal{L} \cdot s}{d} = 1.316 \cdot 10^{-14} \cdot \frac{45 \cdot 10^{-3} \cdot (100 \cdot 10^6)^2 \cdot 1 \cdot 10 \cdot 10^{-2}}{3} =$$

$$= 0.1974 \ \ V/m \ \ \Rightarrow \ \ 105.91 \ \ dB\mu V/m$$

Similarly:

$$|\hat{E}_{CM,max}| = 1.257 \cdot 10^{-6} \frac{|\hat{I}_c| f \mathcal{L}}{d} = 1.257 \cdot 10^{-6} \frac{55 \cdot 10^{-3} \cdot 100 \cdot 10^6 \cdot 1}{3} =$$

$$= 2.3045 \cdot 10^0 \ \ V/m \Rightarrow 127.25 \ \ dB\mu V/m$$

Next, recalling the definition of the antenna factor:

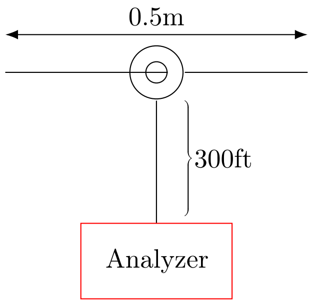
$$AF = \frac{|\hat{E}_{inc}|}{|\hat{V}_{ant}|} \ \ \Rightarrow \ \ AF_{dB/m} = E_{dB\mu V/m} - V_{dB\mu V}$$

3. Consequently:

$$V^{DM}_{dB\mu V} = |\hat{E}_{DM,max}|_{dB\mu V/m} - AF_{dB/m} = 90.91 \ \ dB\mu V$$

$$V^{CM}_{dB\mu V} = |\hat{E}_{CM,max}|_{dB\mu V/m} - AF_{dB/m} = 112.35 \ \ dB\mu V$$

- Problem no.3: *A current probe having* $Z_T = 15\ dB\Omega$ *at 100 MHz measures a current on a 0.5m wire as shown in the picture. The spectrum analyzer is connected to the current probe with a 300 ft length of RG58U coaxial cable, and reads a level of 20* $dB\mu V$ *[The cable loss is 4.5dB/100ft @ 100Mhz]. Determine the electric field in a FCC Class B radiation emission test. Will this device pass the test?*



Solution:

The value measured by the probe in terms of voltage is:

$$V_{ant}(dB\mu V) + \text{Cable loss} = V_{probe} dB\mu V = 20 + \frac{4.5}{100} \times 300 = 33.5\ \ dB\mu V$$

Recall the Ohm's Law:

$$I = V/Z \Rightarrow I_{dB\mu A} = V_{dB\mu V} - Z_{dB\Omega} = 33.5 dB\mu V - 15 dB\Omega = 18.5 dB\mu A$$

This result is the current detected by the probe, and we convert it in natural:

$$18.5\ \ dB\mu A \Rightarrow 8.4139\ \ \mu A$$

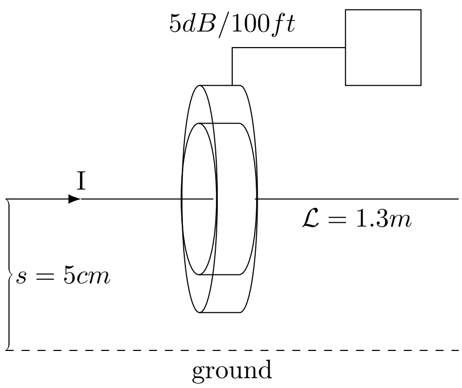
In the end we evaluate the field emitted as half the field of two parallel conductors in common mode at a distance of 3m (FCC test reference value):

$$E = \frac{1}{2} \cdot |\hat{E}_{CM,max}| = \frac{1}{2} \cdot 1.257 \cdot 10^{-6} \cdot \frac{|\hat{I}| f \mathcal{L}}{d} =$$

$$= 1.257 \cdot 10^{-6} \cdot \frac{8.4129 \cdot 10^{-6} \cdot 100 \cdot 10^{6} \cdot (0.5)^2}{3} = 88.125 \cdot 10^{-6} \Longrightarrow 38.90\ \ dB\mu V/m$$

It does respect the FCC limit for Class B devices, that is 43.5 dBμV/m.

- Problem no.4: *With reference to the line above ground shown in the picture, determine the current measured by the monitor probe, knowing that at 300 MHz (a) the voltage measured by a spectrum analyzer (SA) connected to the probe through a 30ft coaxial cable (with cable loss 5dB/100ft @ 300MHz) is 20* $dB\mu V$*, and (b) the probe transfer impedance is* $Z_T = 12 dB\Omega$.



Solution:

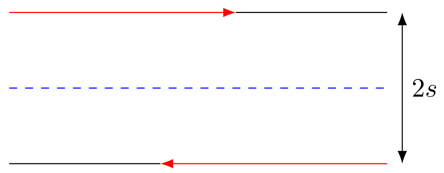
The reasoning of the previous exercise holds true also for this one.

$$V_{probe} = V_{SA} + \text{Cable loss} = 21.5dB \;\; \mu V$$

Hence $I = V_{probe} - Z_T = 9.5 \;\; sB\mu A = 2.985\mu A$.

4.2: *The current probe is the removed and an atenna is placed 10 m far from the signal line. Determine the electric field measured by this antenna, and determine whether these emissions pass FCC class B test (at 300 MHz is 46 dbμV/m)*

Solution:
To solve this problem we need to rely on the principle of images, which let us replace the ground with another wire. This substitution is allowed if the total field respect the boundaries conditions of the ground. Besides, it leads to a solution that is acceptable only for the region of space in which there is the first wire.
However, the equivalent model is the same of the DM evaluation:



- @ d=10m: $E_D = 1.316 \cdot 10^{-14} \frac{If^2 \mathcal{L} s}{d} = 35.97 \;\; \mu V/m = 33.25 \;\; \mu V/m$
- @ d=3m: $E_{D,3m} = E_D + 20 \log_{10} \left(\dfrac{10}{3} \right) = 43.71 \;\; dB\mu V/m.$

It does respect the test limit.

4.3: *Repeat the evaluation at point (2) in the absence of the ground plane.*

Solution:
We compute the field emitted by the wire as the one emitted by two wires in common mode divided by 2...

- @ d=10m: $E_C = \frac{1}{2} 4\pi \cdot 10^{-6} \frac{If\mathcal{L}}{d} = 37.28 dB\mu V/m$
- @ d=3m: $E_{C,3m} = E_C + 20 \log_{10} \left(\frac{10}{3} \right) = 47.74 \;\; dB\mu V/m$

It does not respect the limit.

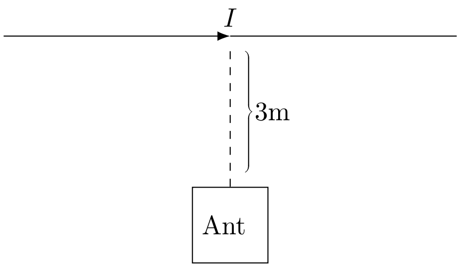
4.4: *For both configurations in (2) and (3), i.e, with and without the ground plane, determine the voltage measured by a spectrum analyser connected ot the antenna through a 50 ft cable (cable loss = 5 dB/ft at 300 MHz) knowing that the antenna factor at 300 MHz is 14 dB/m.*

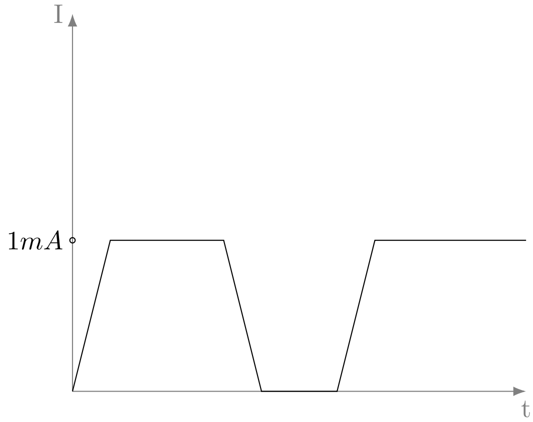
Solution:

$$\begin{cases} AF_{dB/m} = E_{dB\mu V/m} - V_{ant,dB\mu V} \\ V_{SA} = V_{ant} - \text{Cable Loss} \end{cases}$$

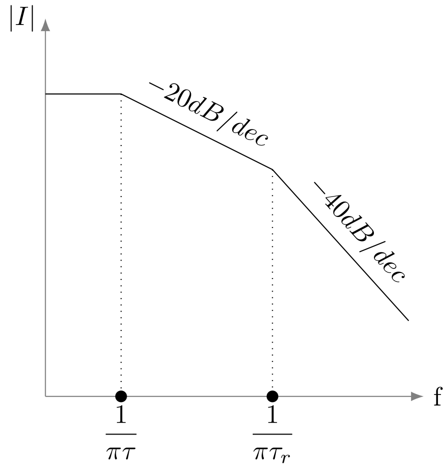
- With the ground: $V_{SA} = E_D(10m) - AF - \frac{5}{100} \times 50 = 16.75 \;\; dB\mu V$
- Without the ground: $V_{SA} = E_C(10m) - AF - \frac{5}{100} \times 50 = 20.78 \;\; dB\mu V$

- Problem no. 5: *The common mode current in a 1 m cable is measured, and consists of a 10 MHz trapezoidal pulse train having a 50% duty cycle and ris/falltimes of 20 ns, as shown in the picture. The maximum of I is 1mA. The radiated emissions of this cable are measured at a distance of 3m parallel to the wire using an antena that has an AF of 8dB at 30MHz and 13dB at 100 MHz. Draw the envelope of the emission as measured on the spectrum analyer between 30 and 100 MHz.*





Solution:
Observe the spectrum of current I:



Exploiting the following definitions we will evaluate the frequencies of the poles and the values of the current corresponding to the ends of the frequency interval we care about:

Maximum:	$\frac{2A\tau}{T} = 2A\delta = 60\ \ dB\mu A\ \ (1\ \ mA)$
Period:	$T_0 = \frac{1}{f_0} = 100ns$
Pulse width:	$\tau = \delta \cdot T_0 = 50ns$

Hence:

$$f_1 = \frac{1}{\pi\tau} = 6.37\ \ MHz \qquad f_2 = \frac{1}{\pi\tau_r} = 15.91\ \ MHz$$

As we can easily see, the frequency we're interested in falls in the third region, in which the decrease is of -40dB/dec.
We will exploite an important and inherent property of the logarithmic plots: there is a linear relation to evaluate the current at a specific frequency knowing the value and the corresponding frequency of another point, and the decrease/increase in between. For istance:

$$I(f_2) = I(f_1) - 20\log_{10}\left(\frac{f_2}{f_1}\right)$$

Since $I(f_1) = 60\ \ dB\mu A$, then $I(f_2) = 52.05\ \ dB\mu A$. Similarly:

- @ 30 MHz: $I(f_3) = I(f_2) - 40\log_{10}\left(\frac{f_3}{f_2}\right) = 41.03\ \ dB\mu A$
- @ 100 MHz: $I(f4) = 21.115\ \ dB\mu A$

Now, recalling that for only on ìe wire, the field is

$$E_{CM/2} = 2\pi \cdot 10^{-7} I \frac{f\mathcal{L}}{d}$$

Hence, in decibel we have:

- @ 30 MHz: $E(f_3) = 20\log_{10}\left(2\pi \cdot 10^{-7}\dfrac{f_3 \times 1m}{3m}\right) = 57\ \ dB\mu V/m$
- @ 100 MHz: $E(f_4) = 20\log_{10}\left(2\pi \cdot 10^{-7}\dfrac{f_4 \times 1m}{3m}\right) = 46.54\ \ dB\mu V/m$

In the end we can compute the envelope of the emissions measured by the spectrum analyzer by considering the AF spectrum:

$$V(f_3) = E(f_3) - AF(f_3) = 49 dB\mu V \qquad V(f_4) = E(f_4) - AF(f_4) = 33.54\ \ dB\mu V$$

Hence the sloped is:

$$slope = \frac{V(f_4) - V(f_3)}{\log_{10}\left(\frac{f_4}{f_3}\right)} = -29.57\ \ dB/dec$$

9 Antenna Parameters

- **Antenna pattern**: three dimensional plot that describes the directional properties of an antenna. The fundamental term is the *normalized radiation intensity*, $F(\theta, \phi)$:

$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{max}}$$

  For the Hertzian dipole, $F(\theta, \phi) = \sin^2 \theta$. The plot resembles a doughnut and $F_{max} = 1$ in the broadside direction. Usually is measured in decibel.

- the antenna is said to be "fairly directed", when most of the energy radiated is focused in the main lobe. The enegy irradiated in the side and back lobes is considered wasted.

- **Pattern solid angle**: defines an equivalent cone over which all the radiationof the actual antenna is concerned with equal intensity.

$$\Omega_p = \iint_{4\pi} F(\theta, \phi) d\Omega \quad [sr]$$

- **Isotropic Antenna**: is an antenna with $F(\theta, \phi) = 1$ in all directions and $\Omega_p = 4\pi$.

- **3-dB bandwidth**: it is defined as the angular width of the main lobe between the angles (θ_1, θ_2) at which the magnitude is equal to half of its peak value: $\beta = \theta_2 - \theta_1$.

- **Directivity**:

$$D = \frac{F_{max}}{F(\theta, \phi)} = \frac{1}{\frac{1}{4\pi} \iint_{4\pi} F(\theta, \phi) d\Omega} = \frac{4\pi}{\Omega_p}$$

  Note that for an isotropic antenna D=1.
  Moreover, D can be also obtained as:

$$D = \frac{S_{max}}{S_{ave}} \quad \wedge \quad S_{ave} = S_{iso}$$

  So, the directivity is defined as the ratio between the maximum power emitted by an antenna and the power by an isotropic antenna.

  For an Hertzian Dipole:

$$D = 1.5 \Rightarrow D_{dB} = 1.76 dB$$

- **Radiation Efficiency**: Of the total power supplied to the antenna (P_t), some is dissipated as heat (P_{loss}) in the antenna while the remaining is actually emitted (P_{rad}). The efficiency:

$$\xi = \frac{P_{rad}}{P_t} = \frac{R_{rad}}{R_{rad} + R_{loss}}$$

- **Gain**: defined as

$$G = \frac{4\pi R^2 S_{max}}{P_t}$$

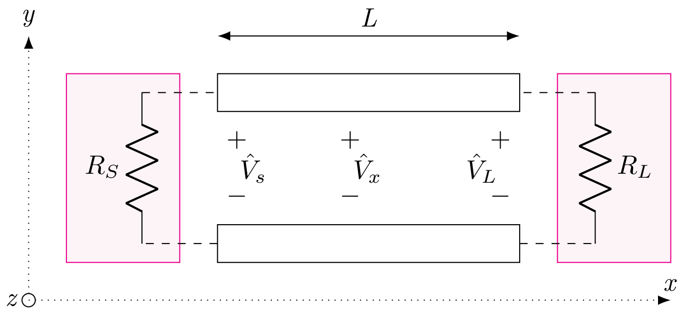
  Observe that there is a relationship between the Gain and the Directivity:

$$G = \xi \cdot D = \frac{P_{rad}}{P_t} \cdot D$$

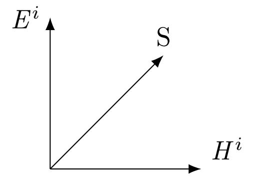
10 Radiated Susceptibility

- **Radiated susceptibility** is due to intentional or unintentional radiation emitted by high-power transmitter or by nearby electrical devices; or it is due to randomic phenomena with pulse characteristics.
- The EUT is **tested** in anechoic environment, in which we use a RF generator with an amplifier to simulate the radiation. The RS test procedure is the following:
  1. Field calibration in the abscence of the EUT to determine the antenna feeding conditions assuming specific field levels where the EUT will be placed;
  2. Place the EUT;
  3. For each frequency: we let the radiation reach the EUT and we verify that it does not exhibit malfunctions.

  Usually the device itself is immune or schielded, but the radiation could couple with cable hearnesses. During the test, cable hearnesses must be exposed at least for 1m from the EUT - after 3m they are uncoupled thanks to the use of ferrites. Moreover, they should be lifted 10 cm far from the ground. As for the EUT, its position must be changed to test it when all its surfaces are parallel to the calibration plane. The cable must be replaced as well when we turn the EUT. Besides, this procedure must be repeated for horizzontal and vertical polarization of the antenna. It's a long procedure.
- **Field to wire Coupling** - introduction



The generic fields configuration of the incident wave:



Note that the propagation direction ($\vec{S}$) is not better specified with respect to the axis in order to include every possible situation.
Now, the components which give rise to the EMI are:

- the electric field transverse to the line direction, that is directed as y:

$$\hat{E}_t^i = \hat{E}_y^i$$

- the magnetic field that is normal to the plane of the wires, that is in the opposite direction of z:

$$\hat{H}_n^i = -\hat{H}_z^i$$

The surroinding medium is assumed homogeneous and nonferromagnetic. For two parallel wires with radius r_w and spacing s, the inductance and capacitance per unit length are:

$$l = \frac{\mu_0}{\pi} \ln\left(\frac{s}{r_w}\right)$$

$$c = \frac{1}{v^2 l} = \frac{\pi\epsilon_0\epsilon_r}{\ln\left(\frac{s}{r_w}\right)}$$

Focus on the normal component of H. The Faraday's Law states that a varying magnetic flux interlinked to a closed circuit gives rise to an electrical current, whose verse respect the thumb rule. The response of the circuit consists of an *induced current* which flows in the opposite direction of the first one, and that generates another magnetic flux compensating the variation of the original flux. Faraday's Law mathematical expression allow to modelize the source of the response like a voltage source.

$$emf(x) = -\jmath\omega \int_S \hat{B}_n^i \cdot ds = -\jmath\omega\mu_0 \Delta x \int_0^s \hat{H}_n^i(x, y) \cdot dy$$

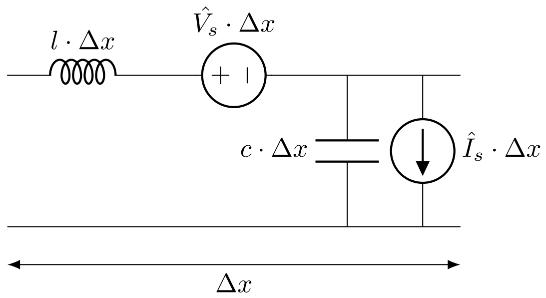
In general the function H is a function depending on the longitudinal coordinate x, and the vertical one, y (z doesn't change).
Do consider an infintesimal segment (Δx) of the transmission line. Taking into account capacitance and inductance per unit length, we have a closed circuit. Remember that we use voltage sources with same direction of the induced current.
The voltage source for this segment is given computing $|emf|/\Delta x$:

$$\hat{V}_s(x) = \jmath\omega\mu_0 \int_0^s \hat{H}_n^i(x, y) dy$$

In a similar way, with a dual analysis, we derive that the **transverse component of E** gives rise to an induced current directed like $-y$. The expression is:

$$\hat{I}_s(x) = \jmath\omega c \int_0^s \hat{E}_t^i(x, y) dy$$

We can obtain this result also with another reasoning: the transverse electrice field induces a voltage ($\hat{V}_E = \int_0^s \hat{E}_t^i(x, y)dy$) in series with the per unit capacitance within the wires (impedance: $1/(\jmath\omega c\Delta x)$). The complete equivalent circuit is:



Analyzing the structure, imposing the KVL and KVC, we have that:

$$\begin{cases} \hat{V}(x + \Delta x) - \hat{V}(x) = -\jmath\omega l \Delta x - \hat{V}_s \Delta x & KVL \\ \hat{I}(x + \Delta x) - \hat{I}(x) = -\jmath\omega c \Delta x \hat{V}(x) - \hat{I}_s \Delta x & KVC \end{cases}$$

We divide for Δx and we impose that $\Delta x \to 0$:

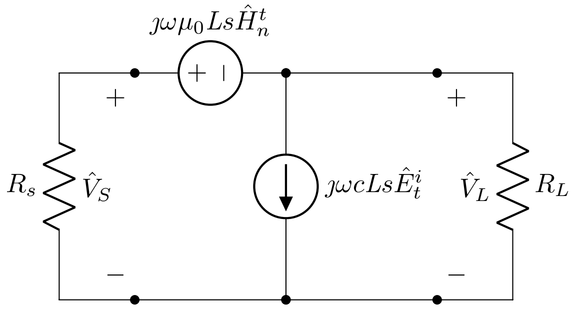
$$\begin{cases} \dfrac{d\hat{V}(x)}{dx} + \jmath\omega l \hat{I}(x) = -\hat{V}_s = -\jmath\omega\mu_0 \int_0^s \hat{H}_n^i(x, y) dy \\ \dfrac{d\hat{I}(x)}{dx} + \jmath\omega c \hat{V}(x) = -\hat{I}_s = -\jmath\omega c \int_0^s \hat{E}_t^i(x, y) dy \end{cases}$$

To estimate the coupling the approximated solution is sufficient.

Approximated Model: for many case of practical interest the lenght is electrically small at the frequency of interest ($L \ll \lambda$). Under this assumption we can consider lumped parameters by using one section to represent all the line and we replace Δx with L.

Moreover, we can neglect the p.u.l. inductances and capacitances so long as the terminal impedances are not extreme values such as short or open circuits.
Now, assuming that the wire separation is way smaller than the wire length (so it's also electrically small), the fields do not vary appreciably across the wire section. Therefore, the integrals for the sources evaluation can be replaced with the wire separation s multiplied by the interfering component. Since we're considering the product with L, we define A the area of the circuit. We obtain:

$$\begin{cases} \hat{V}_s \cdot L = \jmath\omega\mu_0 \hat{H}_n^i \cdot A \\ \\ \hat{I}_s \cdot L = \jmath\omega c \hat{E}_t^i \cdot A \end{cases}$$



We exploit the superposition theorem to compute the voltages at the terminals.

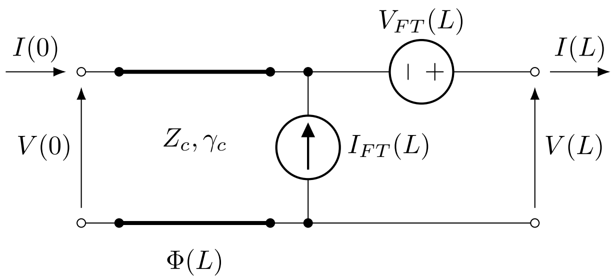
- Important observations.

  1. the field-to wire coupling is proportional to the frequency, to the loop area ($L \cdot$ s) and it can change with the orienttion of the line with repect to the radiation.
  2. in the special case of a matched line with an end-fire excitation:

$$\hat{V}_{induced} = \jmath\omega\mu_0 \hat{H}_n^i \cdot A = \jmath\omega\mu_0 \left(\frac{\hat{E}}{\eta_0}\right) \cdot A = \jmath\omega\frac{\hat{E}}{c_0} \cdot A$$

$$\hat{I}_{induced} = \jmath\omega c \hat{E}_t^i \cdot A = \jmath\omega\frac{\hat{E}}{c_0 Z_c} \cdot A = \frac{\hat{V}_{induced}}{Z_c}$$

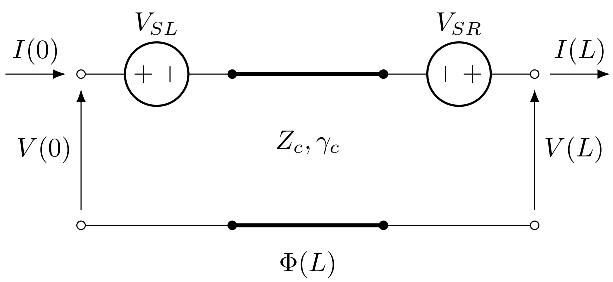
- **Exact solution: equivalent circuit of Taylor.**



Without the assumptions considered for the approximated model, we can cast the equivalent circuit at the end of the transmission line as shown in the picture aside.

$$\begin{bmatrix} \hat{V}(L) \\ \hat{I}(L) \end{bmatrix} = \hat{\Phi}(L) \cdot \begin{bmatrix} \hat{V}(0) \\ \hat{I}(0) \end{bmatrix} + \begin{bmatrix} \hat{V}_{FT}(L) \\ \hat{I}_{FT}(L) \end{bmatrix}$$

- **Exact Solution: equivalent circuit of Agrawal**.



In this second case the voltages sources take on the meaning of line open-end voltages due to wire-field coupling.

- **Shielded Cables**.

  Coaxial cables consists of a concentric shield enclosing an interior wire that is located on the axis of the shield. The intent is to completely enclose a circuit in order to prevent coupling to the terminations from incident fields outside the shield.
  However, the fields can penetrate the shield via diffusion of the current which is induced on the external surface. At this point we introduce some simplifying assumptions...

  For what concerns the *external problem*, we can calculate the current induced I_{SH} by the external field assuming the shield is a perfect conductor: any interaction between interior and exterior is neglected.

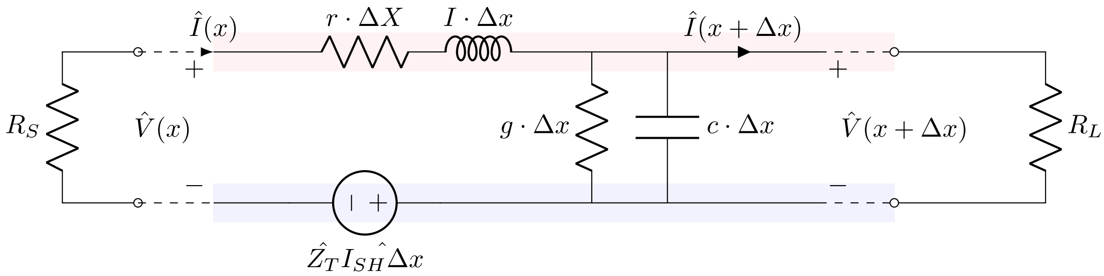
  As for the *internal problem*, the shield current diffuses through the shield wall to give a voltage drop on the interior surface of the shield of:
  $$d\hat{V} = \hat{Z}_T \hat{I}_{SH} dx$$
  where Z_T si the p.u.l surface transfer impedance of the shield.
  For solid shields, Z_T is given by
  $$\hat{Z}_T = r_{dc} \frac{\hat{\gamma} \cdot t_{SH}}{\sinh(\hat{\gamma} \cdot t_{SH})} \qquad (\Omega/m)$$
  where:

P.u.l. Shield Resistance in DC:	$r_{dc} = \dfrac{1}{\sigma 2\pi r_{SH} t_{SH}}$
Thickness:	t_{SH}
Propagation constant:	$\hat{\gamma} = (1 + \jmath)/\delta$
Skin depth:	$\delta = 1/\sqrt{\pi f \mu_0 \sigma}$

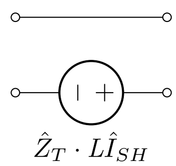
  Analyzing the impedance expression:

  – For $t_{SH} \ll \delta$, the transfer impedance tends to 1;
  – For $t_{SH} > \delta$, the impedance decreases with decreasing skin depth(i.e with increasing frequencies)

  Since the voltage drop along the interior surface of the shield acts as a voltage source, we can sketch the following scheme for an infinitesimal line section:



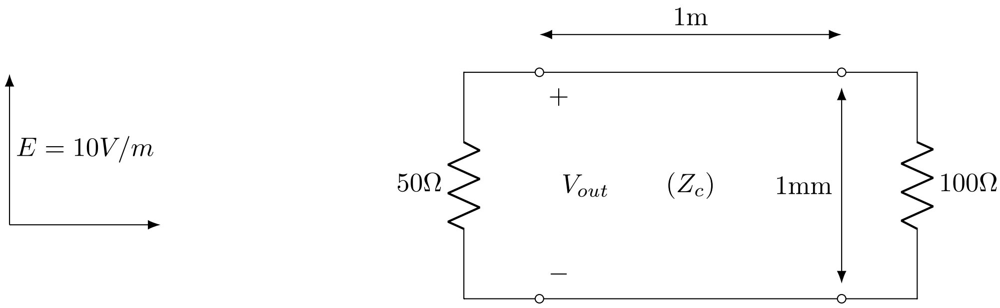
  The lower wire is the interior surface of the shield while the upper one represents the interior wire. For an electricaly short line we can approxiamte the solution by lumping the source and ignoring the p.u.l. parameters of the inner wire-shield circuit. We obtain the following central segment:



For a given shield current, I_{Sh}, the Larger is Z_T the larger is the interference induced at the terminations of the interior circuit. From the EMC viewpoint, it is desirable that a coaxial cable exhibits a transfer impedance as low as possible in the frequency range of interest.

11 Practical Lecture: Radiated Susceptibility

- Problem no. 1: *A 100Mhz, 10V/m uniform plane wave is propagating parallel to an air-filled two-wire transmission line as shown in the picture. The electric field is in the plane of the two wire. COmpute the magnitude of the voltage induced across the 50Ω load.* $Z_c = 300\Omega$.



Solution:

We need the magnetic component to apply the superposition theorem:

$$|H| = \frac{|E|}{\eta_0} = 26.5 \ \ mA/m$$

And we need the capacitance to have all the parameters of the current source:

$$Z_c = \frac{1}{c_0 \cdot C} \Longrightarrow C = \frac{1}{c_0 \cdot Z_c} = \frac{1}{3 \cdot 10^8 \cdot 300} = 11.1 \ \ pF/m$$

According to the equation obtained in the course of the lecture, we can compute the sources:

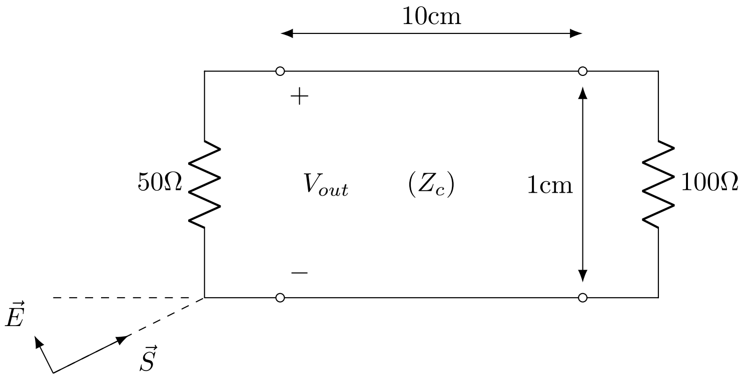
$$\hat{V}_s \mathcal{L} = \jmath \omega \mu_0 H s \mathcal{L} = \jmath 20.9 \ \ mV$$

$$\hat{I}_s \mathcal{L} = \jmath \omega C E s \mathcal{L} = \jmath 69.7 \ \ \mu A$$

Hence we get:

$$\hat{V}_{ount} = -\frac{R_L}{R_R + R_L} \hat{V}_s \mathcal{L} - \frac{R_L R_R}{R_L + R_L} \hat{I}_s \mathcal{L} = -\jmath 9.3 mV$$

- Probelm no. 2: *A 100 MHz, 10 V/m unifrom plane wave is incident on a two-wire line as shown in the picture. Determine the induced voltage V if the cable has a p.u.l. capacitance of* $50pF/m$. *The incident angle is* $\theta = 30$.



Solution:

Similarly to what we did in the previous problem:

1. We need the magnetic component that is normal to the plane of the wires:
   $H^n = {}^E/_{\eta_0} = 26.5 \ \ mA/m$
2. We nedd the transverse component of the electric field with respect to the vertical axis of the tansmission line:
   $$E^t = E\cos\theta = 8.66 \ \ V/m$$
3. Now we are able to compute the sources:
   $$\hat{V}_s\mathcal{L} = \jmath\omega\mu_0 H^n s\mathcal{L} = \jmath 20.9 \ \ mV$$
   $$\hat{I}_s\mathcal{L} = \jmath\omega C E^t s\mathcal{L} = \jmath 0.272 \ \ \mu A$$
4. Finally, the voltage V is
   $$\hat{V}_L = -\frac{R_L}{R_L + R_R}\hat{V}_s\mathcal{L} - \frac{R_L R_R}{R_R + R_L}\hat{I}_s\mathcal{L} = -\jmath 17.3 \ \ mV$$

- Poblem no. 3: *Special Cases: endfire incident, assuming matched loads, so* $R_L = R_R = Z_c$

Solution:

Since $v = 1/\sqrt{\mu\epsilon}$ and $\eta = \sqrt{\mu/\epsilon}$:

$$\hat{V}_s\mathcal{L} = \jmath\omega\mu_0 H s\mathcal{L} = \jmath\omega\mu_0 {}^E/_{\eta_0} s\mathcal{L} = \jmath\omega\tfrac{E}{c_0}s\mathcal{L}$$
$$\hat{I}_s\mathcal{L} = \jmath\omega C E s\mathcal{L} = \jmath\omega E s\mathcal{L}\frac{1}{c_0 \cdot Z_c}$$

Solving the equiavlent circuit, neglecting as always the p.u.l. parameters, we obtain:

$$\hat{V}_L = -\frac{\hat{V}_s\mathcal{L}}{2} - \frac{Z_c}{2}\hat{I}_s\mathcal{L} = -\frac{\jmath\omega E s\mathcal{L}}{2c_0} - \frac{\jmath\omega E s\mathcal{L}}{2c_0} = -\jmath\omega\frac{E s\mathcal{L}}{c_0}$$
$$\hat{V}_R = +\frac{\hat{V}_s\mathcal{L}}{2} - \frac{Z_c}{2}\hat{I}_s\mathcal{L} = \frac{\jmath\omega E s\mathcal{L}}{2c_0} - \frac{\jmath\omega E s\mathcal{L}}{2c_0} = 0$$

The opposite result would be obtained if the wave impinges the right terminal.

- <u>Problem no. 4</u>: *A 1 m shield cable is illuminated by a 1 MHz incident unifrom plane wave. The shield is composed of 16 belts with 4 wires per belt of braid wires having radii of 2.5 mils. The weave angle is 30°. The shield interior radius is 35 mils. Determine the net shield resistance (dc).*
Determine the surface transfer impedance of the shield at 1 MHz.
The interior circuit si terminated in 300 and 50Ω resistors. Determine the voltages induced across the loads if the current induced on the exterior of the shield is 31.5 mA.

  a) 2.5 mils correspond to $(2.5 \times 0.0254)mm = 63.5\mu m$. So we have wires having diameters of 0.127 mm. The resitance of each wire is

$$R_w = \frac{\mathcal{L}}{\sigma \pi r_w^2} = 1.36\Omega$$

  Remember tha the σ for cupper wires is $58 \cdot 10^6 S/m$.
  In DC, the shield resistance is:

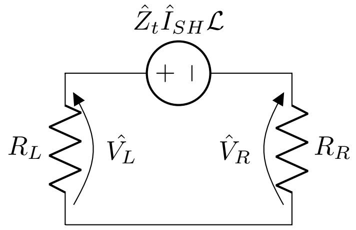
$$R_{DC} = \frac{R_w}{B \cdot w \cdot \cos(\theta_w)} = 24.6m\Omega$$

  b) The formula for the surface impedance of the shield is

$$\hat{Z}_T = R_{DC} \cdot \frac{\hat{\gamma} 2r_w}{\sinh(\hat{\gamma} 2r_w)} = 19.2e^{-64,3^\circ} \quad m\Omega$$

  Where $\hat{\gamma} = \frac{1+\jmath}{\delta}$ and $\delta = \frac{1}{\sqrt{\pi f \sigma \mu_0}} = 66\mu m$

  c) The equivalent circuit is derived and analyzed:



  $R_L = 300\Omega$ $\quad R_R = 50\Omega$ $\quad I_{SH} = 31.5mA$

  Hence:

$$\hat{V}_L = \frac{R_L}{R_R + R_L} \hat{Z}_T \hat{I}_{SH} \mathcal{L} = 0.225 - \jmath 0.468 \ \ V \hat{V}_R = -\frac{R_R}{R_R + R_L} \hat{Z}_T \hat{I}_{SH} \mathcal{L} = -37.46 + \jmath 77.95 \ \ V$$

  Note: since the direction of the shield current is not specifiec we should point out that we should care only about the magnitude of the two voltages.

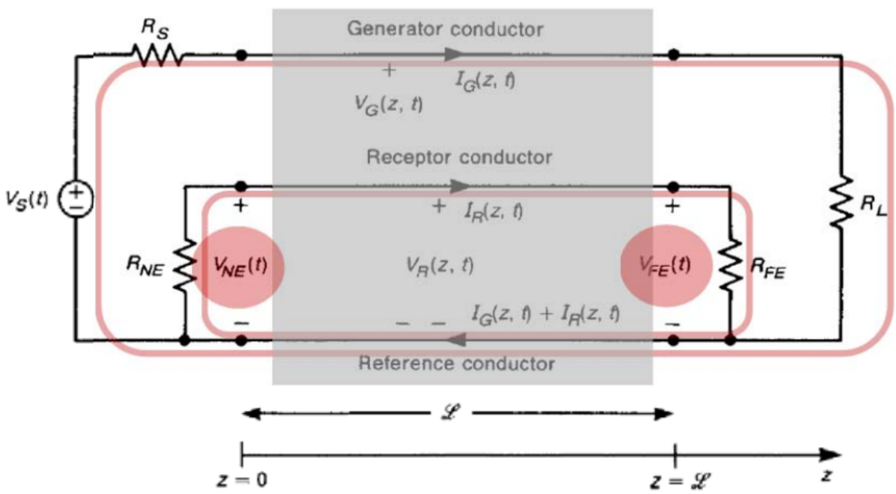
12 Crosstalk

- A quick overview of the conclusions...

Conclusions

1. At low frequencies, xtalk grows linearly (+20dB/decade):
   - CAP coupling dominates for high-impedance terminations;
   - IND couplig dominates for low-impedance terminations;
   - CI coupling is frequency independent and dominates only at very low frequencies.
2. At high frequencies, the line resonances shape the f-response.
3. In Time domain, crosstalk manifests itself as the time derivative of the surce signal (for slow signals).
4. Crosstalk suppression:
   - shielding avoids capacitive xtalk and limits inductive xtalk (if the shield is grounded at both ends)
   - twisting avoids inductive xtalk (capacitive xtalk is suppressed only with balanced terminations)

- **Crosstalk** (or *xtalk*) refers to the unintended electromagnetic couplig between wires an PCB lands that are in close proximity. In other words, it's a near-field EM coupling phenomenon between circuits. There are no standards developed for communication cables, and there are no EMC standards foreseen for crosstalk measurement and compliance verification with limit levels.
- Do consider the **three-conductor lines**: there is a reference wire which closes the circuits related to the generator and to the receptor.
  The current and voltage associated with the generator (G) circuit will generate electromagnetic fields that interact with the receptior (R) circuit.



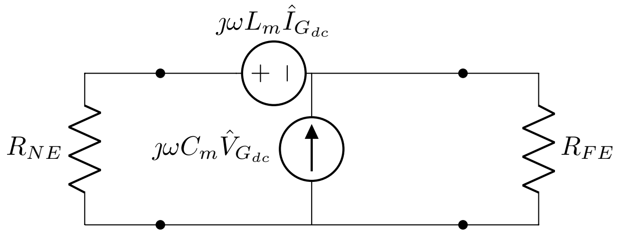
;

As we can see from the picture, we define $V_{NE}(t)$ the near end voltage and V_{FE} the far end one, with respect to the signal source. Obviously we could switch to the frequency domain considering the phasors of these functions: $\hat{V}_{NE}(\jmath\omega)$ or $\hat{V}_{FE}(\jmath\omega)$.

- **Elementary Explanation**. First of all, we do some simplifying assumptions:
  - Low frequency (line is short with respect to wavlength, $\mathcal{L}/\lambda \ll 1$)
  - Weak coupling (wires are sufficiently separated)

  Netx, we will divide the analysis in steps...

  1. Generator circuit alone:
  $$\hat{I}_{G_{DC}} = \frac{1}{R_S + R_L}\hat{V}_S \quad \wedge \quad \hat{V}_{G_{DC}} = \frac{R_L}{R_S + R_L}\hat{V}_s$$
  2. Inductive coupling (IND): the generator current generates a magnetic field whose flux will couple with the "victim" circuit ($L_m = l_m \cdot \mathcal{L}$). There is a closed-form expression of mutual inductance l_m but it does exist only for simple geometries. Otherwise, numerical computation is required. In general:
  $$l_m = \frac{\mu_0}{2\pi}\ln\left(\frac{d_G d_R}{d_{GR} r_{w0}}\right)$$
  3. Capacitive coupling (CAP): electric potential difference induces charges proportional to the capacitance between the two circuits ($C_m = c_m \cdot \mathcal{L}$). Similarly to IND:
  $$c_m = \frac{l_m}{\nu^2(l_G l_R - l_m^2)}$$
  4. We apply the superposition theorem of inductive and capacitance effects, and we evaluate the terminal voltages in the victim circuit.



  $$\begin{cases} \hat{V}_{NE} = \dfrac{R_{NE}}{R_{NE} + R_{FE}}\jmath\omega L_m \hat{I}_{G_{dc}} + \dfrac{R_{NE}R_{FE}}{R_{NE} + R_{FE}}\jmath\omega C_m \hat{V}_{G_{dc}} \\ \\ \hat{V}_{FE} = -\dfrac{R_{FE}}{R_{NE} + R_{FE}}\jmath\omega L_m \hat{I}_{G_{dc}} + \dfrac{R_{NE}R_{FE}}{R_{NE} + R_{FE}}\jmath\omega C_m \hat{V}_{G_{dc}} \end{cases}$$
  5. Manipulating the equations is possible to isolate the transfer ratios $\hat{V}_{FE}/\hat{V}_S$ (or $\hat{V}_{NE}/\hat{V}_S$).

	IND	CAP
NE	$M_{NE}^{IND} = \frac{R_{NE}}{R_{NE} + R_{FE}}\frac{L_m}{R_S + R_L}$	$M_{NE}^{CAP} = \frac{R_{NE}R_{FE}}{R_{NE} + R_{FE}}\frac{R_L C_m}{R_S + R_L}$
FE	$M_{FE}^{IND} = -\frac{R_{FE}}{R_{NE} + R_{FE}}\frac{L_m}{R_S + R_L}$	$M_{FE}^{CAP} = M_{NE}^{CAP}$

  So, in electrically short structure the crosstalk increases linearly with frequency. Moreover, one contribution can be dominant over the other: for istance, inductive coupling prevails if $M^{IND} > M^{CAP}$.

At the near end (NE):

$$\frac{R_{FE}R_L}{L_m/C_m} < 1$$

At the far end (FE):

$$\frac{R_{NE}R_L}{L_m/C_m} < 1$$

Note that for a homogenous medium $L_m/C_m = Z_{CG}Z_{CR}$.
These new parameters are defined as the characreristic impedances of each circuit in the presence of the other:

$$Z_{CG} = \sqrt{\frac{l_G}{c_G + c_m}} \qquad Z_{CR} = \sqrt{\frac{l_R}{c_R + c_m}}$$

They differ from the expression of the classic characteritic impedance for the presence of the mutual capacitance. Which coupling is prevailing?

- The *inductive* couling (IND) dominates for terminations impedances that are low with respect to the circuit characteristic impedances:

  Small terminations impedances ⇒ large current ⇒ large magnetic induction
- On the other hand, the *capacitive* coupling (CAP) dominates for large terminations impedances:

  Large terminations impedances ⇒ large voltage ⇒ large electric induction

6. Coupling via common return conductor. The voltage drop due to *ohmic losses* acts as a source for the victim circuit -

$$\hat{V}_0 \simeq R_0\hat{I}_G \simeq R_0\hat{I}_{G_{dc}} = \frac{R_0}{R_S + R_L}\hat{V}_S$$

Hence, we have another "contribution" to take into account, indeed:

$$\frac{\hat{V}_{NE}^{CI}}{\hat{V}_S} = M_{NE}^{CI}, \quad M_{NE}^{CI} = \frac{R_{NE}}{R_{NE} + R_{FE}} \cdot \frac{R_0}{R_S + R_L}$$

$$\frac{\hat{V}_{FE}^{CI}}{\hat{V}_S} = M_{FE}^{CI}, \quad M_{FE}^{CI} = \frac{R_{FE}}{R_{NE} + R_{FE}} \cdot \frac{R_0}{R_S + R_L}$$

Note that it's frequency independent.

- Conclusion - the coupling between two circuits can be summarized as follows:

$$\begin{cases} \dfrac{\hat{V}_{NE}}{\hat{V}_S} = \jmath\omega(M_{NE}^{IND} + M_{NE}^{CAP}) + M_{NE}^{CI} \\ \dfrac{\hat{V}_{FE}}{\hat{V}_S} = \jmath\omega(M_{FE}^{IND} + M_{FE}^{CAP}) + M_{FE}^{CI} \end{cases}$$

- In the time domain the crosstalk appears to be the derivate over time of the source signal!!
The condition under which this statemente holds true is the following:

$$rise/fall\ time > 10 \cdot T_D$$

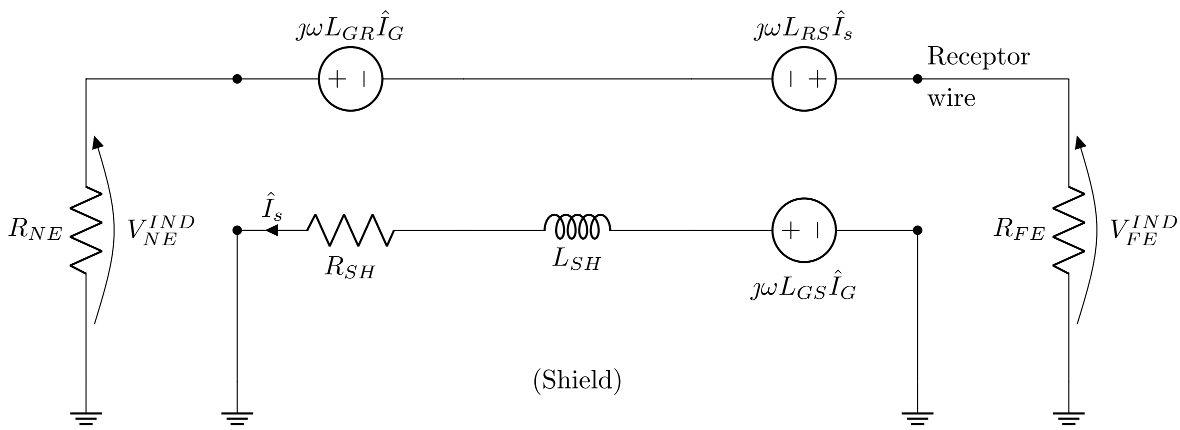
It means that the signal is slow, and $T_D = \mathcal{L}/v$.

- **Shielded wires** are used to avoid capacitive coupling. Indeed, if $V_{shield} = 0$, both c_{RS} and c_{GS} are connected to ground and *we neglect them.* In other words, there is no coupling between R and G provided that the shield is connected to the ground at least at one end.

  Moreover, if the shield is grounded at both ends, *also inductive coupling is reduced*!! This is possible because a return current I_s can flow back along the shield, giving rise to a magnetic flux Ψ_S that counter-acts the flux Ψ_G due to the generator wire loop.

Before continuing, note that with the shield connected to the ground we can separate the structure into three sub-circuits: the generator loop, the receptor/victim circuit and that one replacing the shield.
As we did before, we show this analysis dividing it into different steps:

1. Current in the generator circuit:
$$\hat{I}_G = \frac{1}{R_S + R_L}\hat{V}_S$$
2. Current in the shield circuit:
$$\hat{I}_S = \frac{\jmath\omega L_{GS}}{R_{SH} + \jmath\omega L_{SH}}\hat{I}_G$$
3. NE/FE voltages in the receptor circuit:



$$V_{NE}^{IND} = \frac{R_{NE}}{R_{FE} + R_{NE}}\jmath\omega(L_{GR}\hat{I}_G - L_{RS}\hat{I}_S) = \frac{R_{NE}}{R_{FE} + R_{NE}}\frac{\jmath\omega L_{GR}R_{SH} + \omega^2(L_{GS}L_{RS} - L_{GR}L_{SH})}{R_{SH} + \jmath\omega L_{SH}}\hat{I}_G$$

Now, since:
$$L_{GS} = L_{GR} \qquad L_{RS} = L_{SH}$$

We derive the simplified expressions of these quantities:
$$\begin{cases} \hat{V}_{NE}^{IND} = \dfrac{R_{NE}}{R_{NE} + R_{FE}} \cdot \jmath\omega L_{GR}\hat{I}_G \cdot \dfrac{R_{SH}}{R_{SH} + \jmath\omega L_{SH}} \\ \hat{V}_{FE}^{IND} = -\dfrac{R_{FE}}{R_{NE} + R_{FE}} \cdot \jmath\omega L_{GR}\hat{I}_G \cdot \dfrac{R_{SH}}{R_{SH} + \jmath\omega L_{SH}} \end{cases}$$

Where the effect of the shield depends on the frequency. Note that the
$$f_{SH} = \frac{R_{SH}}{2\pi L_{SH}}$$

If:

- $f < f_{SH}$: $\frac{R_{SH}}{R_{SH}+\jmath\omega L_{SH}} \approx 1$
- $f > f_{SH}$: $\frac{R_{SH}}{R_{SH}+\jmath\omega L_{SH}} \approx \frac{R_{SH}}{\jmath\omega L_{SH}}$

- **Twisted Wires**. It's the dual technique with respect to the shielded wire. We can consider the circuit as a sequence of half-twisted loops. The approximations that we need are the same assumed for the shielded wire, the same valid from the paragraph named "*Elementary explanation*".

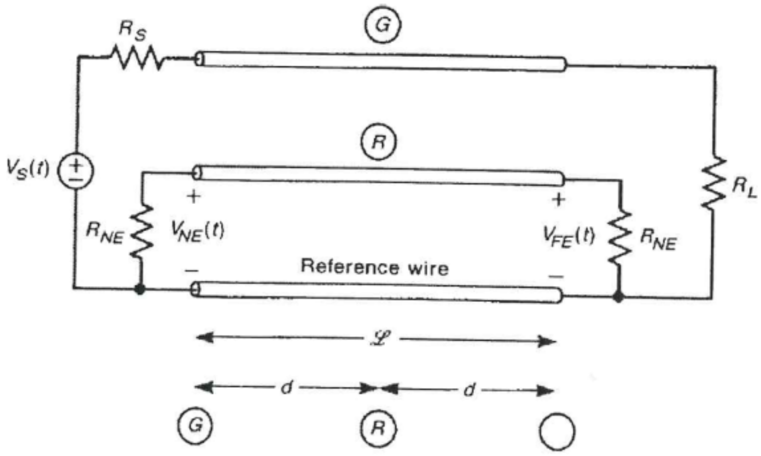
  The magnetic flux due to the generator wire current threads the loop of the TWP, inducing emfs in each loop. However, since the loops alernate in polarity, the induced emfs tend to cancel out in adjacent loops! The worst case occurs for an odd number of loops.

Since this solution is dual with respect to the shielded wire, we expect to have a reduction of inductive coupling, while the capactive is unaltered.

However, it does exist a condition that allow the suppression of capacitive coupling as well: the terminations impedances are balanced.

13 Practical Lecture: Xtalk

- Problem no. 1: *For the ribbon cable shown in picture, assume the total mutual inductance and total mutual capacitance to be* $L_m = 1\mu H$ *and* $C_m = 250pF$*. If* $V_s(t)$ *is a 1 MHz sinusoid of magnitude 1 V, calculate the magnitude of the far-end crosstlak if the termination impedances are* $R_S = 50\Omega, R_L = 50\Omega, R_{NE} = 100\Omega$ *and* $R_{FE} = 100\Omega$*. Determine the near-end and far-end inductive and capacitive coupling coefficients.*

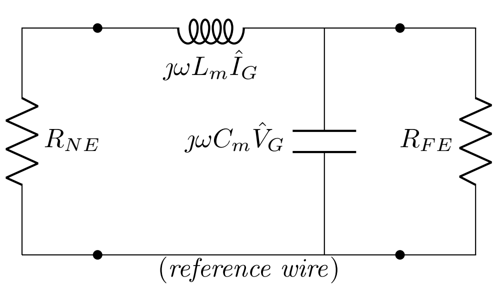


Solution:

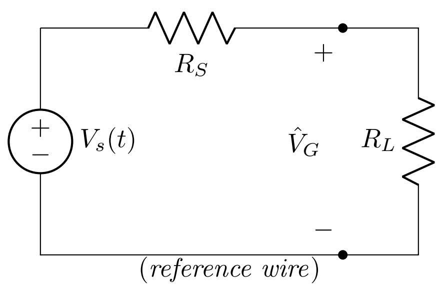
As we studied in the theory we can exploit the superposition theorem, which is applied taking into account three contributions: the inductive coupling, the capacitive coupling and the ohmic losses of the resistances. In first approximation we neglect the last one.
It's really important to point out that the circuits are electrically small, the wires are sufficiently separated and there is not a big missmatching at terminations.

Under this assumptions we can analyze the equivalent circuits:

Receptor circuit:



Generator circuit:



Hence:

$$\hat{I}_G = \frac{\hat{V}_S}{R_L + R_S} \implies \hat{V}_G = \hat{I}_G \cdot R_L = \frac{R_L}{R_S + R_L}\hat{I}_G$$

Now we determine the explicit expressions of the voltages:

$$\hat{V}_{NE} = \frac{R_{NE}}{R_{NE}+R_{FE}}\jmath\omega L_m \hat{I}_G + \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\jmath\omega C_m \hat{V}_G =$$

$$= \jmath\omega\left(L_m \cdot \frac{1}{R_S+R_L}\cdot\frac{R_{NE}}{R_{NE}+R_{FE}}\right)\hat{V}_S + \jmath\omega\left(C_m\cdot\frac{R_L}{R_L+R_S}\cdot\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\right)\hat{V}_S$$

And:

$$\hat{V}_{FE} = -\frac{R_{FE}}{R_{NE}+R_{FE}}\jmath\omega L_m \hat{I}_G + \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}\jmath\omega C_m \hat{V}_G} =$$

$$= \jmath\omega\left(-L_m \cdot \frac{1}{R_S+R_L}\cdot\frac{R_{FE}}{R_{NE}+R_{FE}}\right)\hat{V}_S + \jmath\omega\left(C_m\cdot\frac{R_L}{R_S+R_L}\cdot\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\right)\hat{V}_S$$

The terms inside the brackets are the coefficients we're aiming to evaluate. The first ones are inductive while the second ones are capacitive.
We have:

$$M_{NE}^{CAP} = M_{FE}^{CAP} = 6.25\cdot 10^{-9}\ \ s$$

$$M_{NE}^{IND} = 5\cdot 10^{-9}\ \ s$$

$$M_{FE}^{IND} = -5\cdot 10^{-9}\ \ s$$

And finally:

$$\hat{V}_{FE} = \jmath\omega(M_{FE}^{IND} + M_{FE}^{CAP})\hat{V}_s = \jmath 7.854\ \ mV$$

- Problem no. 2: *For the ribbon cable of the previous problem, assume the total mutual capacitance to be $L_m = 1\ \ \mu H$ and $C_m = 250\ \ pF$. Suppose that the termination impedances are equal: $R_S = R_L = R_{NE} = R$. Determine the value of R for which the inductive and capacitive coupling contributions are exactly equal.*
  Solution:

$$|M^{IND}| = M^{CAP} \longrightarrow L_m\cdot\frac{1}{R_S+R_L}\cdot\frac{R_{FE}}{R_{NE}+R_{FE}} = C_m\cdot\frac{R_L}{R_S+R_L}\cdot\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}$$

$$\Longrightarrow L_m\cdot\frac{R}{4R^2} = C_m\cdot\frac{R^3}{4R^2} = L_m\cdot\frac{1}{R} = C_m\cdot R \Rightarrow R = \sqrt{\frac{L_m}{C_m}} = 63.25\ \ \Omega$$

  If we imagine to have solved the corresponding inequality of the previous equation (for istance $|M^{IND}| > |M^{CAP}|$), we will obtain that if:

  - $R < 63.25\Omega$ the inductive coupling prevails
  - $R > 63.25\Omega$ the capacitive coupling prevails

(More problems following $\cdots$)

- Problem no. 3: *For the ribbon cable of the first problem suppose that the wires are 28-gauge (7×36) stranded. Determine the common-impedance coupling level fo the near-end crosstalk voltage if the total line length is 3 m and the frequency where this level equals the inductive-capaictive coupling level.*
  Solution:

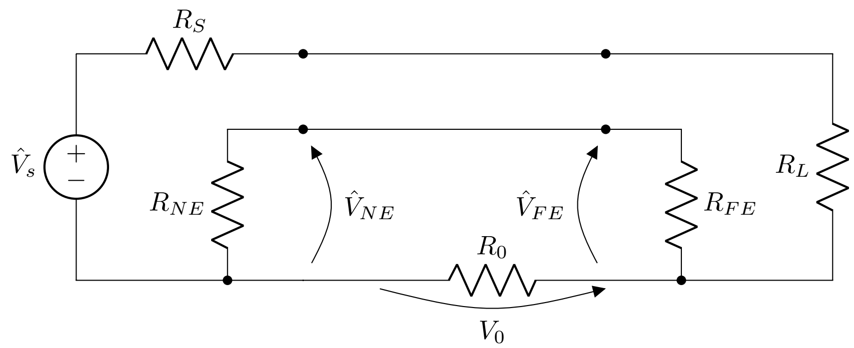
  Looking on a table for the dimension of the AWG

  $$d_w \approx 15 \ \ mil \longrightarrow r_w = 7.15 \quad = 0.19mm$$

  Secondly we need to recall the general formula to compute the resistance of a cupper wire ($\sigma_{Cu} = 58 \cdot 10^6 \ \ S/m$):

  $$R_0 = \mathcal{L} \cdot \frac{1}{\sigma_{Cu} \pi r_w^2} = 0.4537 \ \ \Omega$$

  Now we can analyze the equivalent circuit, calling R_0 the common-impedance of the "return" wire:



  $$\hat{V}_0 = R_0 \hat{I}_G = \frac{R_0 \hat{V}_S}{R_S + R_L + R_0} \simeq \frac{R_0 \hat{V}_S}{R_S + R_L}$$

  Hence, rememberign that $R_{NE} = R_{FE}$:

  $$\hat{V}_{NE} = \left(\frac{R_{NE}}{R_{NE} + R_{FE}} \cdot \frac{R_0}{R_S + R_L} \right) \cdot \hat{V}_S$$

  $$\hat{V}_{FE} = \left(- \frac{R_{FE}}{R_{NE} + R_{FE}} \cdot \frac{R_0}{R_S + R_L} \right) \hat{V}_S = -\hat{V}_{NE}$$

  When IND+CAP is prevailing over the CI? In other words: until which frequencies CI is prevailing?

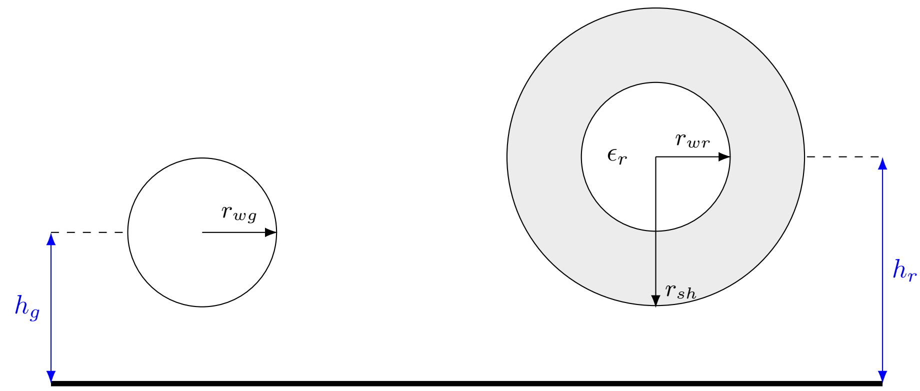
  - @ near-end:

    $$2\pi f_{NE} |M_{NE}^{IND} + M_{NE}^{CAP}| = |M_{NE}^{CI}| \implies f_{NE} \simeq 32.5 \ \ KHz$$

  - @ far-end:

    $$2\pi f_{FE} |M_{FE}^{IND} + M_{FE}^{CAP}| = |M_{FE}^{CI}| \implies f_{FE} \simeq 292.9 \ \ KHz|$$

- Probelm no. 4: *Consider the case of two wires above a ground plane shown in the picture. THe line has parameters of* $l_m = 2\ nH/m,\ c_m = 0.6\ pF/m,\ V_S(t) = 1\cos(\omega \cdot t)\ V,\ f = 1\ MHz,\ \mathcal{L} = 2\ m,\ R_S = 0, R_L = 50\ \Omega,\ R_{NE} = 200\ \Omega,\ R_{FE} = 200\ \Omega$. *A shield is placed around the receptror wire, and is connected to the ground plane only at the near end. Determine the near-end crosstalk voltage. By how much does the shield reduce the crosstalk?.*



Solution:

We know from the theory that if the shield is grounded at least at one end, and it does happen in this case, the capacitive coupling is zero.
Next:

$$\hat{V}_{NE}(shielded) = j167.1\ \ \mu V$$

The attenuation:

$$ATT = 20\log_{10}\left|\frac{M_{NE}^{IND} + M_{NE}^{CAP}}{M_{NE}^{IND}}\right| \simeq 12dB$$

Problem no. 5: *The shield of the previous problem is connected to the ground plane at both ends, and has a per-unit-length resistance of* $1\ \ \Omega/m$ *and per-unit-length self inductance of* $16\ \ \mu H/m$. *Determine the near-end crosstalk voltage.*
Solution:

In general we can compute the near-end voltage as follows:

$$\hat{V}_{NE}^{IND} = \frac{R_{NE}}{R_{FE} + R_{NE}} \jmath\omega L_{GR}\hat{I}_G \frac{R_{SH}}{R_{SH} + \jmath\omega L_{SH}}$$

But, collecting R_{SH} in the last term we obtain:

$$\hat{V}_{NE}^{IND} = \frac{R_{NE}}{R_{FE} + R_{NE}} \jmath\omega L_{GR}\hat{I}_G \frac{1}{1 + \jmath\omega {}^{L_{SH}}/_{R_{SH}}} = \jmath\omega \cdot M_{NE}^{IND} \cdot \frac{1}{1 + \jmath\omega\tau_{SH}}$$

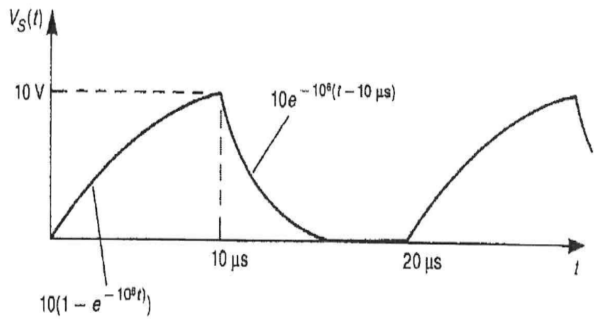
Where $\tau_{SH} = {}^{L_{SH}}/_{R_{SH}} = {}^{l_{SH}}/_{r_{SH}} = 16\ \ \mu s$.
The next step is to evaluate the "cut-off" frequency for the attenuation:

$$f_{SH} = \frac{1}{2\pi\tau_{SH}} \longrightarrow f_{SH} = 9.95kHz$$

Finally we can evaluate the near-end voltage and the attenuation in dB at the working frequency:

- @ 1 MHz: $SF = -20\log_{10}|1 + \jmath\omega\tau_{SH}| = -40\ \ dB$
- Since $1MHz \gg f_{SH}$: $\hat{V}_{NE}^{IND} = \jmath\omega M_{NE}^{IND} \cdot \dfrac{1}{\jmath\omega\tau_{SH}} = 1.67\ \ \mu V$

- Probem no. 6: *Consider the ribbon cable of the first exercise. The total mutual inductances is $L_m = 1\mu H$, and the total mutual capacitance is $C_m = 25pF$. If $R_S = 0$, $R_L = R_{NE} = R_{FE} = 100\ \Omega$, and the pulse waveform shown in the picture below is applied, sketch the time-domain near-end crosstalk and determine the maximum crostalk voltage level.*



Solution:

- f-domain: $\hat{V}_{NE} = \jmath\omega M_{NE}\hat{V}_S$
- time-domain: $V_{NE}(t) = M_{NE} \cdot \frac{dv_s(t)}{dt}$

Hence we only need to apply the formula and we will find the mathematical expression of what is requested:

1. Near-end coefficient:

$$M_{NE} = \frac{R_{NE}}{R_{NE}+R_{FE}} \cdot \frac{L_m}{R_S+R_L} + \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} \cdot \frac{R_L C_m}{R_S+R_L} = 6.25 \cdot 10^{-9}\quad s$$

2. the time derivate of $v_s(t)$:

$$v_s(t) = \begin{cases} 10(1-e^{-10^6 t}), & V \quad 0 < t < 10\mu s \\ 10e^{-10^6(t-10^{-5})}, & V \quad 10\mu s < t < 20\mu s \end{cases}$$

Hence:

$$\frac{dv_s(t)}{dt} = \begin{cases} 10^7 \cdot e^{-10^6 t}, & V \quad 0 < t < 10\mu s \\ -10^7 \cdot e^{-10^6(t-10^{-5})}, & V \quad 10\mu s < t < 20\mu s \end{cases}$$

3. Finally:

$$V_{NE}(t) = \begin{cases} 62.5e^{-10^6 t}, & mV \quad 0 < t < 10\mu s \\ -62.5e^{-10^6(t-10^{-5})}, & mV \quad 10\mu s < t < 20\mu s \end{cases}$$